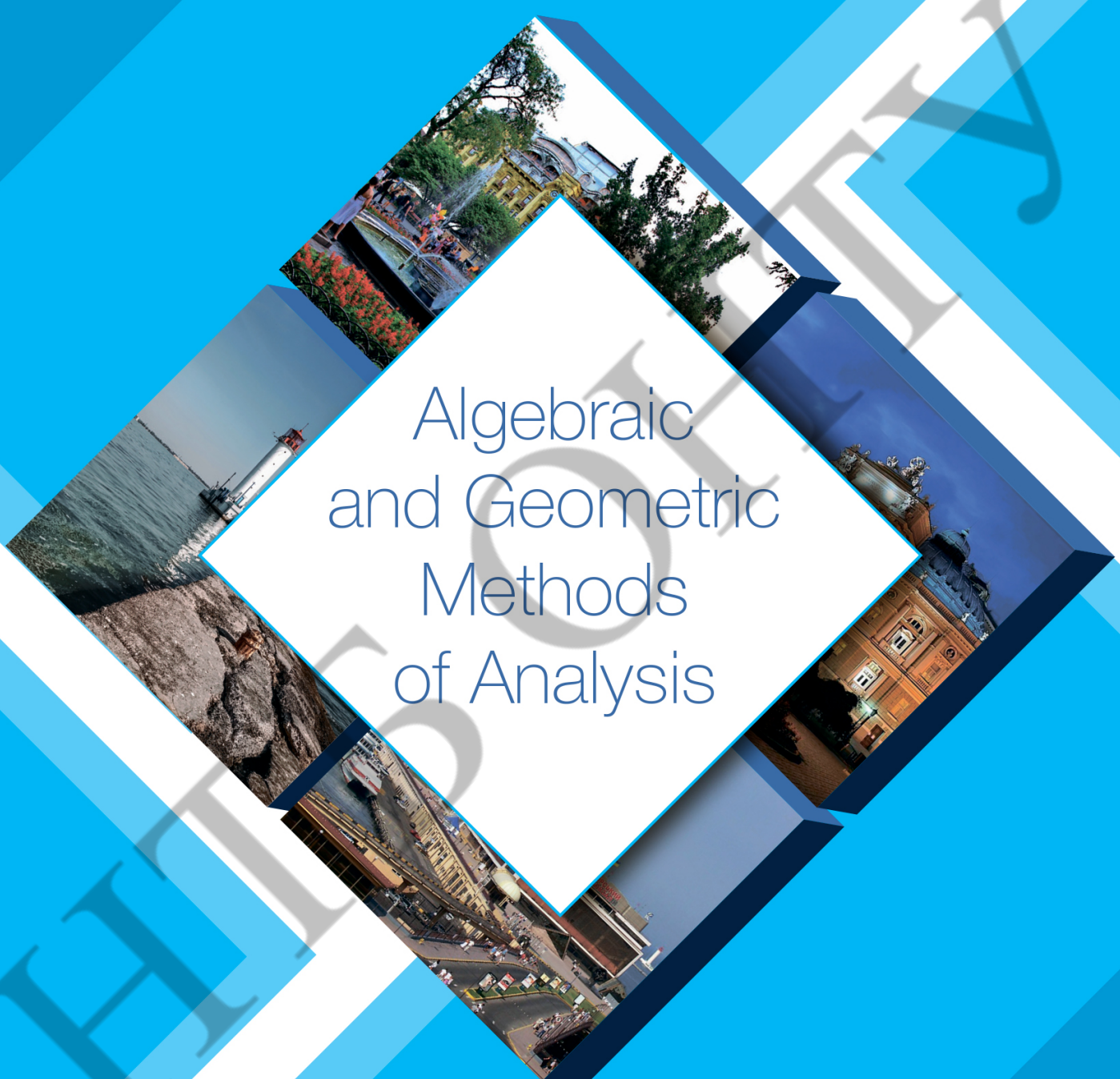


International  
Scientific Conference



Algebraic  
and Geometric  
Methods  
of Analysis

27-30 May 2024  
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

#### LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

#### ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

#### SCIENTIFIC COMMITTEE

- |   |  |
|---|--|
| • <b>Vladimir Balan</b> ( <i>Bucharest, Romania</i> ) | • <b>Volodymyr Lyubashenko</b> ( <i>Kyiv, Ukraine</i> )  |
| • <b>Taras Banakh</b> ( <i>Lviv, Ukraine</i> )        | • <b>Sergiy Maksymenko</b> ( <i>Kyiv, Ukraine</i> )      |
| • <b>Dmytro Bolotov</b> ( <i>Kharkiv, Ukraine</i> )   | • <b>Koji Matsumoto</b> ( <i>Yamagata, Japan</i> )       |
| • <b>Vyacheslav Boyko</b> ( <i>Kyiv, Ukraine</i> )    | • <b>Piotr Mormul</b> ( <i>Warsaw, Poland</i> )          |
| • <b>Yulia Fedchenko</b> ( <i>Odesa, Ukraine</i> )    | • <b>Maryna Nesterenko</b> ( <i>Kyiv, Ukraine</i> )      |
| • <b>Oleg Gutik</b> ( <i>Lviv, Ukraine</i> )          | • <b>Roman Popovych</b> ( <i>Kyiv, Ukraine</i> )         |
| • <b>Olena Karlova</b> ( <i>Chernivtsi, Ukraine</i> ) | • <b>Alexandr Prishlyak</b> ( <i>Kyiv, Ukraine</i> )     |
| • <b>Volodymyr Kiosak</b> ( <i>Odesa, Ukraine</i> )   | • <b>Aleksandr Savchenko</b> ( <i>Kherson, Ukraine</i> ) |
| • <b>Nadiia Konovenko</b> ( <i>Odesa, Ukraine</i> )   |  |

#### ORGANIZING COMMITTEE

- |   |  |
|---|--|
| • <b>Nadiia Konovenko</b> ( <i>Odesa, Ukraine</i> ) | • <b>Bohdan Mazhar</b> ( <i>Kyiv, Ukraine</i> )      |
| • <b>Yuliya Fedchenko</b> ( <i>Odesa, Ukraine</i> ) | • <b>Sergiy Maksymenko</b> ( <i>Kyiv, Ukraine</i> )  |
| • <b>Mykola Lysynskiy</b> ( <i>Kyiv, Ukraine</i> )  | • <b>Alexandr Prishlyak</b> ( <i>Kyiv, Ukraine</i> ) |

In the present talk, we prove that the transformation operator  $\widehat{\mathbb{T}}$  is one-to-one mapping between the sets of the solutions to system (4)–(6) and to system (1)–(3). The application of the operator  $\widehat{\mathbb{T}}$  allows us to conclude that the control system (1)–(3) replicates the controllability properties of the control system (4)–(6) and vice versa. A relation between controls  $u$  and  $v$  is also found. Thus, using obtained results for control system (4)–(6), we obtain the following main results for control system (1)–(3).

**Theorem 3.** *If a state  $w^0 \in \mathring{\mathbb{H}}^1$  is null-controllable with respect to system (1)–(3) in a time  $T > 0$ , then  $w^0 = 0$ .*

**Theorem 4.** *Each state  $w^0 \in \mathring{\mathbb{H}}^1$  is approximately controllable to any target state  $w^T \in \mathring{\mathbb{H}}^1$  with respect to system (1)–(3) in a given time  $T > 0$ .*

All obtained results have been published in [3].

#### REFERENCES

- [1] Larissa Fardigola and Kateryna Khalina. Controllability problems for the heat equation on a half-axis with a bounded control in the Neumann boundary condition. *Math. Control Relat. Fields*, 11(1) : 211–236, 2021.
- [2] L.V. Fardigola. Transformation operators and modified Sobolev spaces in controllability problems on a half-axis. *J. Math. Phys. Anal. Geom.*, 12(1) : 17–47, 2016.
- [3] Larissa Fardigola and Kateryna Khalina. Controllability problems for the heat equation with variable coefficients on a half-axis controlled by the Neumann boundary condition. *J. Math. Phys. Anal. Geom.*, 19(3) : 616–641, 2023.

## On explicit reconstruction of real algebraic maps locally like moment maps

Naoki Kitazawa

(Institute of Mathematics for Industry, Kyushu University)

*E-mail:* n-kitazawa@imi.kyushu-u.ac.jp / naokikitazawa.formath@gmail.com

Real algebraic geometry studies real algebraic varieties. Nash and Tognoli’s theory shows that a smooth closed manifold is realized as the zero set of a real algebraic map and non-singular. It is also well-known that smooth maps between non-singular real algebraic manifolds are approximated by real algebraic maps in general. For related history and terminologies and notions, see [5] for example.

Our study is on explicit construction of real algebraic sets and maps. The  $k$ -dimensional unit sphere  $S^k$  in the  $(k+1)$ -dimensional real affine space  $\mathbb{R}^{k+1}$  and its canonical embedding into higher dimensional real affine spaces (and the compositions with canonical projections) give simplest examples. In general, it is very difficult to give real algebraic sets and important real algebraic maps on them very explicitly. Here, we reconstruct real algebraic maps generalizing the canonical projections of the unit spheres  $S^k \subset \mathbb{R}^{k+1}$  from given regions in the target spaces. We discuss this from the viewpoint of global singularity theory and differential topology of manifolds.

**Theorem 1.** [2] *Let  $l_1, l_2$  and  $n$  be positive integers. Let  $D \subset \mathbb{R}^n$  be an open subset. Let  $\{S_j\}_{j=1}^{l_1}$  be a family of non-singular real algebraic hypersurfaces of  $\mathbb{R}^n$ . Let  $S_j$  be also the zero set of a real polynomial  $f_j$ . We also assume the following.*

- (1) It holds that  $D \cap \bigcup_{j=1}^{l_1} S_j = \emptyset$ , that  $\bar{D} \cap S_j \neq \emptyset$  for  $1 \leq j \leq l_1$  and that  $\bar{D} - D \subset \bigcup_{j=1}^{l_1} S_j$ . For any small open neighborhood  $U_D$  of  $\bar{D}$ ,  $D = U_D \cap \bigcap_{j=1}^{l_1} \{x \mid f_j(x) > 0\}$  and  $\bar{D} = U_D \cap \bigcap_{j=1}^{l_1} \{x \mid f_j(x) \geq 0\}$ .
- (2) Let  $\{i_j\}_{j=1}^{i_0}$  be an increasing sequence of integers such that  $1 \leq i_j \leq l_1$ . Let  $p \in \bigcap_{j=1}^{i_0} S_{i_j} \cap \bar{D}$  and for any increasing sequence  $\{i'_j\}_{j=1}^{i_0+1}$  of integers satisfying  $1 \leq i'_j \leq l_1$  and containing  $\{i_j\}_{j=1}^{i_0}$  as a subsequence,  $p \notin \bigcap_{j=1}^{i_0+1} S_{i'_j}$  hold. Let  $e_j : S_j \rightarrow \mathbb{R}^n$  denote the canonical embedding. Assume that  $\bigcap_{j=1}^{i_0} S_{i_j}$  is an  $(n - i_0)$ -dimensional smooth submanifold of  $\mathbb{R}^n$  with no boundary and let  $e_{\{i_j\}_{j=1}^{i_0}} : \bigcap_{j=1}^{i_0} S_{i_j} \rightarrow \mathbb{R}^n$  denote the canonical embedding. Then the intersection  $\bigcap_{j=1}^{i_0} de_j(T_p S_j)$  and the image  $de_{\{i_j\}_{j=1}^{i_0}}(T_p(\bigcap_{j=1}^{i_0} S_{i_j}))$  of the differential of  $e_{\{i_j\}_{j=1}^{i_0}}$  at  $p$  always agree.
- (3) There exists a map  $m_{l_1, l_2}$  which maps each integer  $1 \leq i \leq l_1$  to an integer  $1 \leq i' \leq l_2$ , which is a surjection to the set of all integers  $1 \leq j \leq l_2$ , and whose restriction to an increasing sequence  $\{i_j\}_{j=1}^{i_0}$  making  $\bigcap_{j=1}^{i_0} S_{i_j} \cap \bar{D}$  a non-empty set is always injective.

Let  $m_{l_2}$  be a map mapping each integer  $1 \leq i \leq l_2$  to a non-negative integer. Let  $m := n + \sum_{j=1}^{l_2} m_{l_2}(j)$ . Then there exist an  $m$ -dimensional non-singular real algebraic manifold  $M \subset \mathbb{R}^{m+l_2}$  and a real algebraic map  $f : M \rightarrow \mathbb{R}^n$  such that  $f(M) = \bar{D}$ , that the image of the singular set is  $\bar{D} - D$ , and that each preimage  $f^{-1}(p)$  ( $p \in \bar{D}$ ) is a single-point set or a product of spheres and at most  $(m - n)$ -dimensional.

We present the case where hypersurfaces  $S_j$  do not intersect and  $l_2 = 1$ . Here  $M := \{(x, y) \in \mathbb{R}^n \times \mathbb{R}^{m-n+1} = \mathbb{R}^{m+1} \mid \prod_{j=1}^{l_1} (f_j(x) - \|y\|^2) = 0\}$ . The map  $f : M \rightarrow \mathbb{R}^n$  is the composition of the canonical embedding into  $\mathbb{R}^{m+1}$  with the canonical projection. In the case  $\bar{D} := D^n$ , the  $n$ -dimensional unit sphere, we have the canonical projection of the unit sphere  $S^m \subset \mathbb{R}^{m+1}$  into  $\mathbb{R}^n$ . This is already done in [1, 3]. In the case the hypersurfaces  $S_j$  do not intersect, we have so-called *special generic* maps presented in [4]. See also [6] for special generic maps in the theory of differential topology of manifolds. Our result reconstructs real algebraic maps locally like moment maps.

As an application, Theorem 2 presents explicit families of real algebraic functions which are compositions of maps into  $\mathbb{R}^2$  obtained through Theorem 1 with the canonical projection ([2]). This is seen as extending some result of [1]: we have studied cases where the hypersurfaces  $S_j$  do not intersect. We define the *Reeb graph*  $W_c$  of a smooth function  $c$  on a closed manifold  $X$ . Let  $\sim_c$  be the following equivalence relation on  $X$ :  $x_1 \sim_c x_2$  if and only if  $x_1$  and  $x_2$  are in a same connected component of a preimage  $c^{-1}(y)$  ( $y \in \mathbb{R}$ ). Let  $q_c : X \rightarrow W_c := X/\sim_c$  denote the quotient map. The vertex set is the set of all points  $v$  such that  $q_c^{-1}(v)$  contain some critical points of  $c$ . See also [7] for example.

**Theorem 2.** [2] In Theorem 1, let  $n = 2$ ,  $l_1 := l_{0,1}$ ,  $\{S_j := S_{0,j}\}_{j=1}^{l_1}$  consist of mutually disjoint circles and  $D := D_0 \subset \mathbb{R}^2$  be a connected and bounded open set surrounded by the disjoint union  $\bigsqcup_{j=1}^{l_{0,1}} S_j$ .

We construct a map  $f := f_{M_0} : M := M_0 \rightarrow \mathbb{R}^2$  by Theorem 1. We consider the function  $f_{0, M_0} := \pi_{2,1} \circ f_{M_0}$  where  $\pi_{2,1} : \mathbb{R}^2 \rightarrow \mathbb{R}$  is the canonical projection. Let  $m \geq 4$  be an integer. For each non-negative integer  $k$ , we can consider some situation enjoying the following and apply Theorem 1.

- (1)  $n = 2$ .  $l_1 := l_{0,1} + k$ .  $l_2 := 2$ .
- (2) The family  $\{S_j\}_{j=1}^{l_1}$  is a family of circles enjoying the following.

- (a) For  $1 \leq j \leq l_{0,1}$ ,  $S_j := S_{0,j}$  and the family  $\{S_{l_{0,1}+j}\}_{j=1}^k$  consists of mutually disjoint and sufficiently small circles in  $\mathbb{R}^2$  centered at points in some circles in  $\{S_j := S_{0,j}\}_{j=1}^{l_1}$ .
- (b) For each  $S_{l_{0,1}+j_2}$  ( $1 \leq j_2 \leq k$ ), there exists a unique circle  $S_{0,j_1}$  such that  $S_{l_{0,1}+j_2} \cap S_{0,j_1} \neq \emptyset$ . Furthermore, the non-empty intersection is a two-point set.
- (3) The open set  $D \subset \mathbb{R}^2$  is the connected and bounded component of the complementary set of  $\bigcup_{j=1}^{l_1} S_j$  in  $\mathbb{R}^2$  which is also a subset of  $D_0$ .

Furthermore, choosing suitable situations, we have a family  $\{f := f_{M_k} : M := M_k \rightarrow \mathbb{R}^2\}_{k=0}^\infty$  of real algebraic maps on  $m$ -dimensional closed and connected manifolds enjoying the following properties.

- (4) The Reeb graph  $W_{f_{0,M_k}}$  of the function  $f_{0,M_k} := \pi_{2,1} \circ f_{M_k}$  collapses to the Reeb graph  $W_{f_{0,M_0}}$ .
- (5) The graphs  $W_{f_{0,M_{k_1}}}$  and  $W_{f_{0,M_{k_2}}}$  are not isomorphic for distinct integers  $k_1$  and  $k_2$ .

The author was a member of the project JSPS Grant Number JP17H06128 and a member of the project JSPS KAKENHI Grant Number JP22K18267. Our study has been conducted under their support. The author works at Institute of Mathematics for Industry (<https://www.jgmi.kyushu-u.ac.jp/en/about/young-mentors/>) and this is closely related to our study. The author is also a researcher at Osaka Central Advanced Mathematical Institute (OCAMI researcher), supported by MEXT Promotion of Distinctive Joint Research Center Program JPMXP0723833165.

## REFERENCES

- [1] N. Kitazawa, *Real algebraic functions on closed manifolds whose Reeb graphs are given graphs*, Methods of Functional Analysis and Topology Vol. 28 No. 4 (2022), 302–308, arXiv:2302.02339, 2023.
- [2] N. Kitazawa, *Natural real algebraic maps of non-positive codimensions with prescribed images whose boundaries consist of non-singular real algebraic hypersurfaces satisfying transversality*, arXiv:2303.10723v4 is submitted to a refereed journal, arXiv:2303.10723v5.
- [3] N. Kitazawa, *Some remarks on real algebraic maps which are topologically special generic maps*, arXiv:2312.10646.
- [4] N. Kitazawa, *A note on cohomological structures of special generic maps*, a revised version is submitted based on positive comments by referees and editors after the third submission to a refereed journal.
- [5] J. Kollár, *Nash's work in algebraic geometry*, Bulletin (New Series) of the American Mathematical Society (2) 54, 2017, 307–324.
- [6] O. Saeki, *Topology of special generic maps of manifolds into Euclidean spaces*, Topology Appl. 49 (1993), 265–293, we can also find at "<https://core.ac.uk/download/pdf/81973672.pdf>" for example.
- [7] O. Saeki, *Reeb spaces of smooth functions on manifolds*, International Mathematics Research Notices, maa301, Volume 2022, Issue 11, June 2022, 8740–8768, <https://doi.org/10.1093/imrn/maa301>, arXiv:2006.01689.

<b>V. Dryuma</b> <i>6D-Riemannian metric associated at the Navier-Stokes equations and its applications</i>	<b>29</b>
<b>K. Eftekharinasab</b> <i>Interplay of Global Implicit Functions and Critical Point Theory in Infinite Dimensional Spaces</i>	<b>32</b>
<b>M. Egwe</b> <i>Spherical Analysis on Fuzzy Lie Groups</i>	<b>32</b>
<b>V. Fedorchuk, V. Fedorchuk</b> <i>On the construction and classification of the common invariant solutions for the <math>(1 + 3)</math>-dimensional Euler-Lagrange-Born-Infeld and homogeneous Monge-Ampere equations</i>	<b>33</b>
<b>B. Feshchenko</b> <i>Normal forms of functions with degenerate critical points on surfaces whose stabilizers are homotopically non-trivial</i>	<b>35</b>
<b>M. R. Formica</b> <i>Nonlinear interpolation of <math>\alpha</math>-Holderian mappings with applications to quasilinear PDEs</i>	<b>36</b>
<b>J.F. Gálvez-Rodríguez, C. Martín-Aguado, M.A. Sánchez-Granero</b> <i>Riemann Integration on a space with a fractal structure</i>	<b>37</b>
<b>I. Gkeneralis</b> <i>Topological rigidity of quoric manifolds</i>	<b>39</b>
<b>O. Gok</b> <i>On the multiplicative order convergence on Banach lattice <math>f</math>-algebras</i>	<b>41</b>
<b>M. Golasiński</b> <i>Octonionic Stiefel manifolds and vector bundles</i>	<b>43</b>
<b>T. Goy</b> <i>Some series involving central binomial coefficients</i>	<b>44</b>
<b>O. Gutik</b> <i>On non-topologizable semigroups</i>	<b>46</b>
<b>H. Chi</b> <i>Existence and non-existence of cohomogeneity one Einstein metrics</i>	<b>48</b>
<b>I. Havrylenko, E. Petrov</b> <i>Stability of vertical minimal surfaces in three-dimensional sub-Riemannian manifolds</i>	<b>50</b>
<b>O. Ye. Hentosh</b> <i>Rational factorization of Lax type flows in the space dual to the centrally extended Lie algebra of matrix super-integro-differential operators</i>	<b>52</b>
<b>O. Hukalov, V. Gordevskyy</b> <i>The some solution of the Bryan-Pidduck equation</i>	<b>54</b>
<b>S. Ivković</b> <i>Disjoint dynamical properties of wedge operators</i>	<b>55</b>
<b>R. Ilemobade, T.G. Jaiyeola</b> <i>On semi-symmetric <math>(\alpha, \beta, \gamma)</math>-inverse quasigroup</i>	<b>55</b>
<b>O. Karlova</b> <i>About Rolewicz theorem on inversion of continuous bijection between <math>F</math>-spaces</i>	<b>57</b>
<b>L. Fardigola, K. Khalina</b> <i>On boundary controllability problems for the heat equation with variable coefficients on a half-axis</i>	<b>58</b>
<b>N. Kitazawa</b> <i>On explicit reconstruction of real algebraic maps locally like moment maps</i>	<b>60</b>
<b>S.D. Koval, R.O. Popovych</b> <i>Group classification of Kolmogorov backward equations with power diffusivity</i>	<b>63</b>