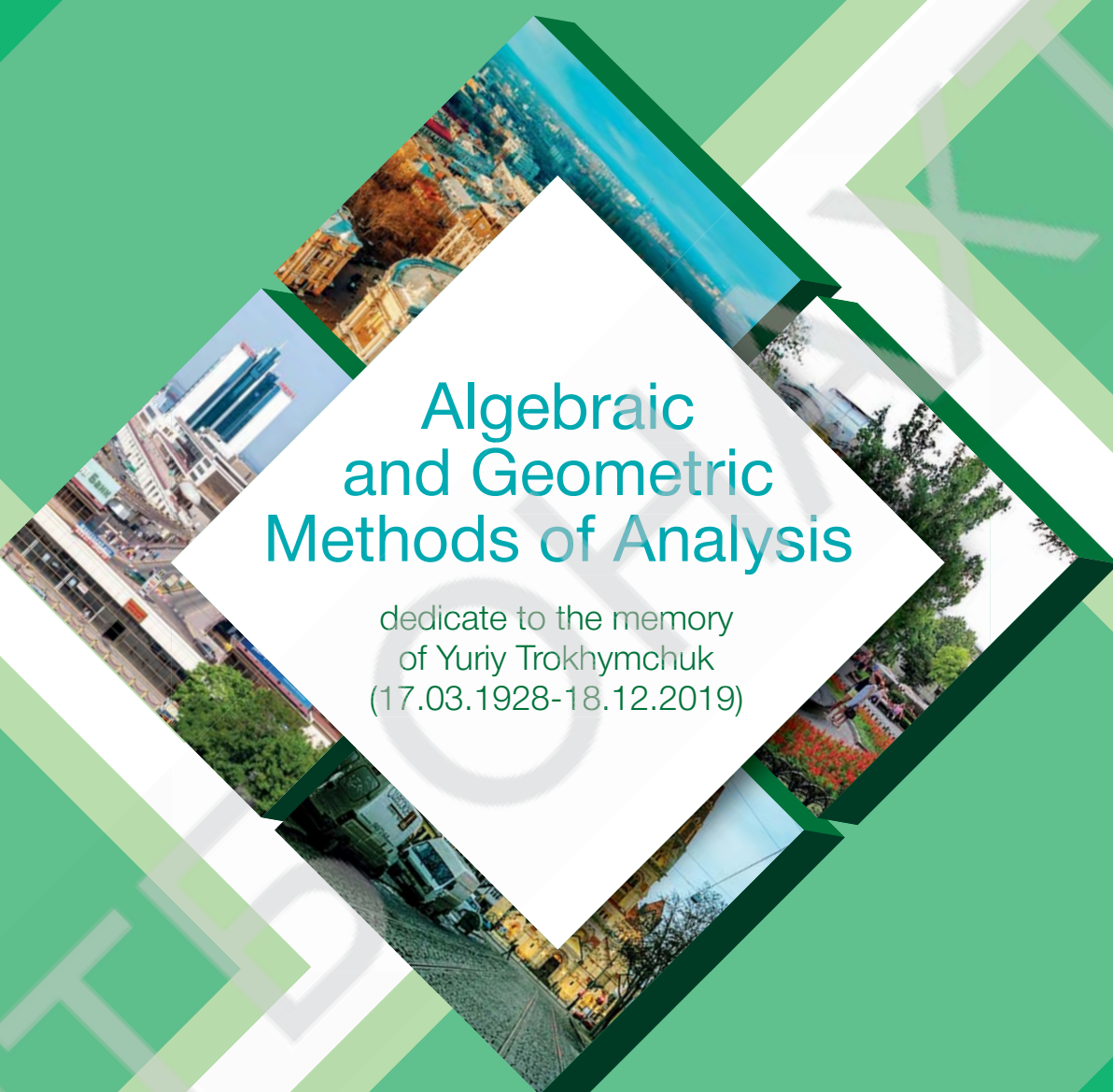


International
Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Problem on extremal decomposition of the complex plane

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Let \mathbb{N} , \mathbb{R} be the sets of natural and real numbers, respectively, let \mathbb{C} be the complex plane, and let $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ be its one-point compactification, $\mathbb{R}^+ = (0, \infty)$. Let $r(B, a)$ be the inner radius of the domain $B \subset \overline{\mathbb{C}}$ relative to a point $a \in B$. The inner radius of the domain B is connected with Green's generalized function $g_B(z, a)$ of the domain B by the relations

$$g_B(z, a) = -\ln|z - a| + \ln r(B, a) + o(1), \quad z \rightarrow a,$$

$$g_B(z, \infty) = \ln|z| + \ln r(B, \infty) + o(1), \quad z \rightarrow \infty.$$

Definition 1. Let $n \in \mathbb{N}$, $n \geq 2$. The system of points $A_n := \{a_k \in \mathbb{C} : k = \overline{1, n}\}$ is called n -ray, if $|a_k| \in \mathbb{R}^+$ for $k = \overline{1, n}$ and $0 = \arg a_1 < \arg a_2 < \dots < \arg a_n < 2\pi$.

Denote $\alpha_k := \frac{1}{\pi} \arg \frac{a_{k+1}}{a_k}$, $\alpha_{n+1} := \alpha_1$, $k = \overline{1, n}$, $\sum_{k=1}^n \alpha_k = 2$.

Problem 2. (V.N. Dubinin [1, 2]) For all values of the parameter $\gamma \in (0, n]$ to show that the maximum of the functional

$$I_n(\gamma) = r^\gamma(B_0, 0) \prod_{k=1}^n r(B_k, a_k),$$

where $B_0, B_1, B_2, \dots, B_n$, $n \geq 2$, are pairwise disjoint domains in $\overline{\mathbb{C}}$, $a_0 = 0$, $|a_k| = 1$, $k = \overline{1, n}$, is attained for the configuration of domains B_k and points a_k which possesses the n -fold symmetry.

In work [1], the above-formulated problem was solved for the value of the parameter $\gamma = 1$ and all values of the natural parameter $n \geq 2$. Namely, it was shown that the following inequality holds

$$r(B_0, 0) \prod_{k=1}^n r(B_k, a_k) \leq r(D_0, 0) \prod_{k=1}^n r(D_k, d_k),$$

where d_k, D_k , $k = \overline{0, n}$, are the poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{(n^2 - 1)w^n + 1}{w^2(w^n - 1)^2} dw^2.$$

In work [3], L.V. Kovalev got its solution for definite sufficiently strict limitations on the geometry of arrangement of the systems of points on a unit circle, namely, for systems of points for which the following inequalities hold

$$0 < \alpha_k \leq 2/\sqrt{\gamma}, \quad k = \overline{1, n}, \quad n \geq 5.$$

In work [4], it was shown that the result by L.V. Kovalev is true for $n = 4$. The solution of this problem for $\gamma \in (0, 1]$ was given in work [5]. Some partial cases of this problem were studied, for example, in [6–10].

For the further analysis, we calculate the quantity

$$I_n^0(\gamma) = r^\gamma(D_0, 0) \prod_{k=1}^n r(D_k, d_k),$$

where $d_k, D_k, k = \overline{0, n}, d_0 = 0$, are, respectively, the poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{(n^2 - \gamma)w^n + \gamma}{w^2(w^n - 1)^2} dw^2.$$

As was shown in [1, 2, 3, 6], the quantity $I_n^0(\gamma)$ takes the form

$$I_n^0(\gamma) = \left(\frac{4}{n}\right)^n \frac{\left(\frac{4\gamma}{n^2}\right)^{\frac{\gamma}{n}}}{\left(1 - \frac{\gamma}{n^2}\right)^{n + \frac{\gamma}{n}}} \left(\frac{1 - \frac{\sqrt{\gamma}}{n}}{1 + \frac{\sqrt{\gamma}}{n}}\right)^{2\sqrt{\gamma}}.$$

Theorem 3. [9] *Let $\gamma \in (1, 2]$. Then, for any different points a_1 and a_2 of a unit circle and any mutually disjoint domains $B_0, B_1, B_2, a_1 \in B_1 \subset \overline{\mathbb{C}}, a_2 \in B_2 \subset \overline{\mathbb{C}}, a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}$, the inequality*

$$r^\gamma(B_0, 0) r(B_1, a_1) r(B_2, a_2) \leq I_2^0(\gamma) \left(\frac{1}{2} |a_1 - a_2|\right)^{2-\gamma}.$$

is true. The sign of equality in this inequality is attained, when the points a_0, a_1, a_2 and the domains B_0, B_1, B_2 are, respectively, the poles and circular domains of the quadratic differential

$$Q(w)dw^2 = -\frac{(4 - \gamma)w^2 + \gamma}{w^2(w^2 - 1)^2} dw^2.$$

Remark 4. Theorem 3 yields the complete solution of the above-posed problem of finding the maximum of product of inner radii of two domains relative to the points of a unit circle on the degree γ of the inner radius of the domain relative to the origin at arbitrary $\gamma \in (0, 2]$, provided that all three domains are mutually non-overlapping domains.

Theorem 5. [9] *Let $n \in \mathbb{N}, n \geq 3, \gamma \in (1, n]$. Then, for any system of different points $A_n = \{a_k\}_{k=1}^n \in \mathbb{C} \setminus \{0\}$ of a unit circle and for any collection of mutually disjoint domains $B_0, B_k, a_0 = 0 \in B_0 \subset \overline{\mathbb{C}}, a_k \in B_k \subset \overline{\mathbb{C}}, k = \overline{1, n}$, the following inequality holds*

$$r^\gamma(B_0, 0) \prod_{k=1}^n r(B_k, a_k) \leq \left(\sin \frac{\pi}{n}\right)^{n-\gamma} \left(I_2^0\left(\frac{2\gamma}{n}\right)\right)^{\frac{n}{2}}.$$

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