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- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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Recent progress in Iwasawa theory of knots and links

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We briefly survey joint works with Ryoto Tange, Hyuga Yoshizaki, and Sohei Tatenno.

Twisted Iwasawa invariants of knots [1]. Let K be a knot in S^3 with $\pi_K = \pi_1(S^3 - K)$ and let $X_n \rightarrow X = S^3 - K$ denote the $\mathbb{Z}/n\mathbb{Z}$ -cover for each $n \in \mathbb{Z}_{>0}$. Let p be a prime number and let $m \in \mathbb{Z}$ with $p \nmid m$. Let $\rho : \pi_K \rightarrow \mathrm{GL}_N(O_p)$ be a representation over a finite extension O_p of the p -adic number field $\mathbb{Q}_p$ and let $\Delta_\rho(t)$ denote the twisted Alexander polynomial. Then we have the following.

Theorem 1. *Let (K, p, m, ρ) be as above. Then there exists some $\lambda, \mu, \nu \in \mathbb{Z}$ such that for any $n \gg 0$, $|H_1(X_{mp^n}, \rho)_{\mathrm{tor}}| = p^{\lambda n + \mu p^n + \nu}$ holds. (We have $\mathrm{Nr} \Delta_\rho(T) \doteq p^\mu (\lambda + p(\text{lower terms}))$ in $\mathbb{Z}_p[[T]]$.)*

For each (K, p, ρ) , there exists some m such that $\lambda/[O_p] : \mathbb{Z}_p] = \deg \Delta_\rho(t)$. Hence for each K , there exists some (p, ρ, m) such that λ coincides with the genus of K .

For each (p, K, ρ) , μ 's and λ 's determine whether $\Delta_\rho(t)$ is monic in $O_p[t]$ and whether K is fibered.

Example 2. (1) The λ 's of the lifts $\rho_{\mathrm{hol}}^\pm : \pi_K \rightarrow \mathrm{SL}_2(O)$ of the holonomy representation of the figure eight knot $K = 4_1$.

(2) For any SL_2 -representations of the twist knots $J(2, 2k)$ ($k \in \mathbb{Z}$), we have $\mu = 0$. We may expect that if $k \neq 0, \pm 1$, then there exists some ρ of $J(2, 2k)$ with $\mu > 0$.

Weber's class number problem for knots [2]. Weber's class number problem for number fields is mostly unsolved for 200 years. Yoshizaki [3] recently pointed out that the sequence of the class numbers converges in the ring of p -adic integers $\mathbb{Z}_p$. In the knot theory side, we obtain the following.

Theorem 3. *Let K be a knot in S^3 and let p be a prime number. Then the sizes of the p -torsion subgroups of $H_1(X_{p^n}; \mathbb{Z})$ converges in $\mathbb{Z}_p$. The limit value is given by the roots of unity that are close to the roots of the Alexander polynomial $\Delta_K(t)$.*

Example 4. The limit values for the torus knot $T_{a,b}$ ($a, b \in \mathbb{Z}$; coprime) and the twist knot $J(2, 2k)$ ($k \in \mathbb{Z}$).

Iwasawa invariants of the $\mathbb{Z}_p^d$ -covers of links [4]. Cuoco–Monsky gave a variant of the Iwasawa class number formula for $\mathbb{Z}_p^d$ -extensions of number fields and pointed out the existence of the term $O(1)$. In our side, we have the following.

Theorem 5. *Let L be a d -component link in a rational homology 3-sphere M and let $Y_n \rightarrow X = M - L$ denote the $\mathbb{Z}/n\mathbb{Z}^d$ -cover. Then there exists some λ, μ such that the size of p -torsion subgroup of $H_1(Y_n, \mathbb{Z})$ is given by $p^{p^{(d-1)n}(\mu p^n + \lambda n + O(1))}$, where $O(1)$ is the Bachmann–Landau notation. If M is an integral homology 3-sphere, then the $\mathbb{Z}_p^d$ -cover is Greenberg, namely, $O(1)$ is a constant.*

Example 6. The values μ , λ , and $O(1)$ of Solomon's link 4_1^2 and the twisted Whitehead link W_{2k-1} ($k \in \mathbb{Z}$). We have a link with $O(1) \neq 0$ and a link with any $\mu \in \mathbb{Z}_{\geq 0}$.

REFERENCES

- [1] Ryoto Tange and Jun Ueki. *Twisted Iwasawa invariants of knots*, preprint, 2022. arXiv:2203.03239
- [2] Jun Ueki and Hyuga Yoshizaki. *On Weber's class number problem for knots*, preprint, 2022.
- [3] Hyuga Yoshizaki, *A New Continued Fraction Expansion and Weber's Class Number Problem*, in preparation, 2020. arXiv: 2010.06399
- [4] Sohei Tateno and Jun Ueki. *Iwasawa invariants of $\mathbb{Z}_p^d$ -covers of links*, in preparation, 2022.

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