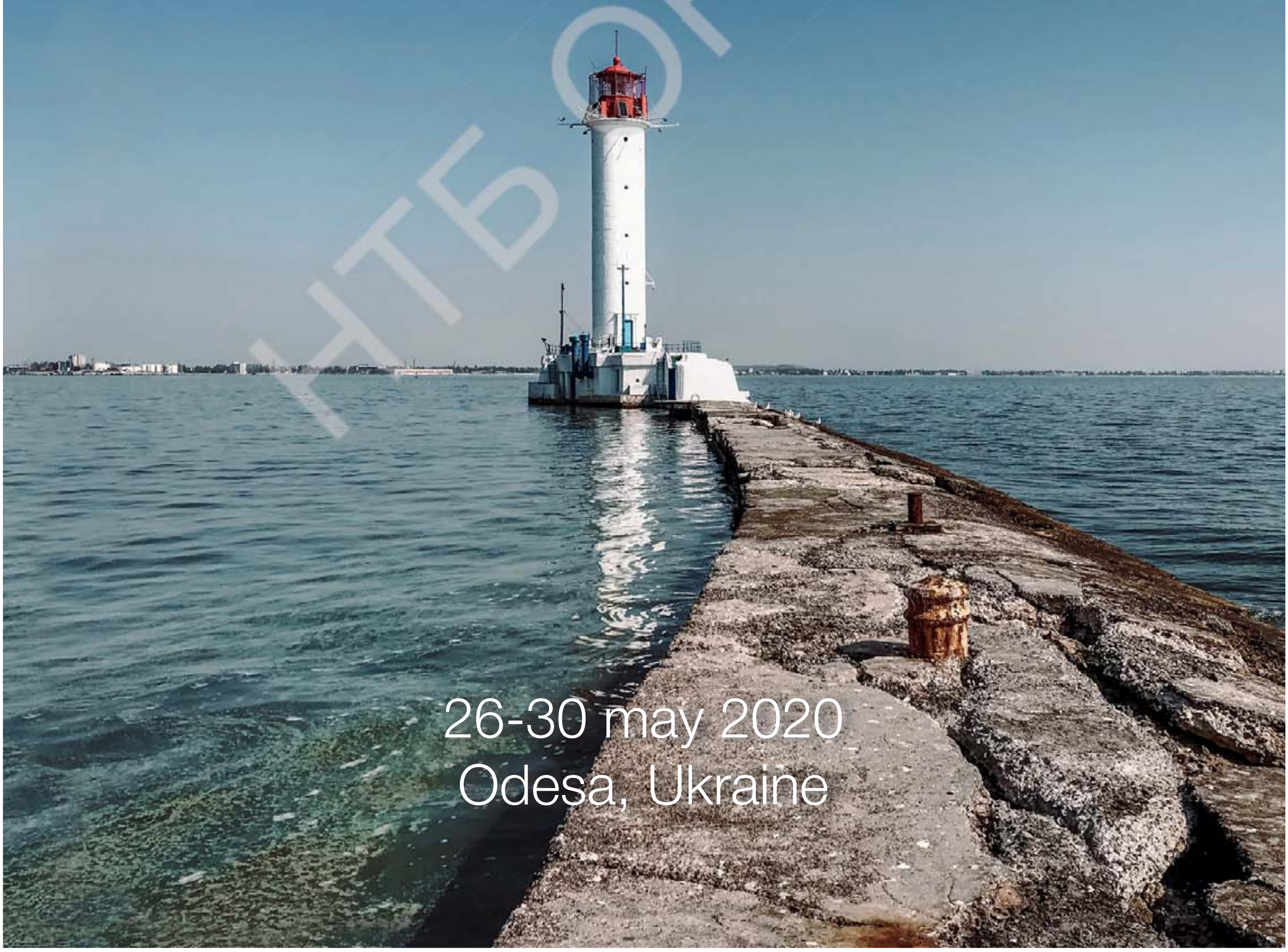
International
Scientific Conference

Algebraic and Geometric Methods of Analysis



26-30 may 2020
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Odessa I. I. Mechnikov National University
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

Program Committee

Chairman: Prishlyak A.
(*Kyiv, Ukraine*)
Balan V.
(*Bucharest, Romania*)
Banakh T.
(*Lviv, Ukraine*)
Bolotov D.
(*Kharkiv, Ukraine*)
Borysenko O.
(*Kharkiv, Ukraine*)
Cherevko Ye.
(*Odesa, Ukraine*)
Fedchenko Yu.
(*Odesa, Ukraine*)
Karlova O.
(*Chernivtsi, Ukraine*)
Kiosak V.
(*Odessa, Ukraine*)
Kirillov V.
(*Odesa, Ukraine*)
Konovenko N.
(*Odesa, Ukraine*)
Lyubashenko V.
(*Kyiv, Ukraine*)
Maksymenko S.
(*Kyiv, Ukraine*)
Matsumoto K.
(*Yamagata, Japan*)
Mormul P.
(*Warsaw, Poland*)
Mykhailyuik V.
(*Chernivtsi, Ukraine*)
Plachta L.
(*Krakov, Poland*)
Pokas S.
(*Odesa, Ukraine*)
Polulyakh E.
(*Kyiv, Ukraine*)
Sabitov I.
(*Moscow, Russia*)
Savchenko A.
(*Kherson, Ukraine*)
Sergeeva A.
(*Odesa, Ukraine*)
Shelekhov A.
(*Tver, Russia*)
Volkov V.
(*Odesa, Ukraine*)
Zarichnyi M.
(*Lviv, Ukraine*)

Administrative committee

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Svytyy I., Dean of the Faculty of Computer Systems and Automation.

Organizing commitee

Kirillov V.
Konovenko N.
Fedchenko Yu.
Maksymenko S.
Cherevko Ye.
Osadchuk E.
Prus A.

Functors and fuzzy metric spaces

Aleksandr Savchenko
(Kherson State Agrarian University, 23 Stretenska str., 73006 Kherson, Ukraine)
E-mail: savchenko.o.g@ukr.net

Mykhailo Zarichnyi
(Ivan Franko National University of Lviv, 1 Universytetska str., 79000 Lviv, Ukraine)
E-mail: zarichnyi@yahoo.com

H. Toruńczyk and J. West [11] considered the construction of (completed) infinite iteration of the hyperspace functor and established some of its geometric properties. A counterpart of this construction for the superextension functor was investigated in [12]. Later, V. Fedorchuk [3] introduced a general notion of perfectly metrizable functor and obtained generalizations of results from [12].

Having in mind increasing interest to the fuzzy metric spaces we are going to extend the notion of perfectly metrizable functor over the class of fuzzy metric spaces in the sense of George and Veeramani [5].

Let X be a set, $*\colon [0,1]\times[0,1]\to[0,1]$ be a continuous t-norm. A GV-fuzzy metric on X is a mapping $m\colon X\times X\times\mathbb{R}_+\to(0,1]$ satisfies the following conditions for all $x,y,z\in X$, $s,t\in\mathbb{R}_+$:

(1GV) $m(x,y,t)>0$;
(2GV) $m(x,y,t)=1$ if and only if $x=y$;
(3GV) $m(x,y,t)=m(y,x,t)$;
(4GV) $m(x,z,t+s)\geq m(x,y,t)*m(y,z,s)$;
(5GV) the function $m(x,y,-)\colon\mathbb{R}_+\to[0,1]$ is continuous.

We will consider the class of (closed to) normal functors in the category **Comp** of compact Hausdorff spaces and continuous maps. For any such a functor F, there exists a natural transformation $\eta\colon 1_{\mathbf{Comp}}\to F$.

Suppose that to each compact fuzzy metric space $(X,m_X,*)$ a fuzzy metric $m_{F(X)}$ on the space $F(X)$ is assigned (with respect to the same triangular norm $*$). We will make the following assumptions:

(1) If $f\colon(X,m_X)\to(Y,m_Y)$ is an isometric embedding, then so is

$$F(f)\colon(F(X),m_{F(X)})\to(F(Y),m_{F(Y)}).$$

(2) The map $\eta_X\colon(X,m_X)\to(F(X),m_{F(X)})$ is an isometric embedding.
(3) $\mathrm{diam}(X,m_X)=\mathrm{diam}(F(X),m_{F(X)})$.

We suppose now that the mentioned functor F is a functorial part of a monad (F,η,ψ). Then we additionally require that the following holds.

(4) The map $\psi_X\colon(F^2(X),m_{F^2(X)})\to(F(X),m_{F(X)})$ is nonexpanding.

The condition in the definition from [3] that concerns the preservation of uniform continuity (this is equivalent to the preservation of (ε,δ)-continuity) does not have a unique counterpart in the case of fuzzy metric spaces, as in the latter case there are different notions of uniform continuity (see, e.g., [6, 4]).

In the talk, we discuss the question of completion of the metric direct limits of the form

$$F^+(X)=\varinjlim\{X\to F(X)\to F^2(X)\to\dots\},$$

with bonding maps $F^n(X)\to F^{n+1}(X)$ taken from the set $\{F^j(\eta_{F^{n-j}(X)})\mid 0\leq j\leq n\}$. Remark that the completion of fuzzy metric spaces does not necessarily exist [7].

Next, we consider the question of embedding of the (completions of the) spaces $F^+(X)$ in the spaces of the form
$$F^\omega(X) = \varprojlim\{F(X) \leftarrow F^2(X) \leftarrow F^3(X) \leftarrow \dots\}$$
As an example, we consider the hyperspace functor ([8]; see also [1]). Another example is the functor of idempotent measures; its fuzzy metrization is constructed in [2]. Recall that the idempotent measures are counterparts of the probability measures in idempotent mathematics, i.e., the part of mathematics in which at least one of arithmetic operations in $\mathbb{R}$ is replaced by an idempotent operation (e.g., max or min).

In the paper [10], a metrization of functors of finite degree is constructed. The considerations of this paper are significantly extended in [1], where the so called ℓ^p-metrics are defined on the sets of the form $F(X)$, where F is a functor with finite supports on the category of sets.

References

[1] T. Banakh, V. Brydun, L. Karchevska, M. Zarichnyi, The ℓ^p-metrization of functors with finite supports, Preprint, arXiv:2004.02017

[2] V. Brydun, A. Savchenko, M. Zarichnyi. Fuzzy metrization of the spaces of idempotent measures. *European Journal of Mathematics*, 6 (1) : 98–109, 2020.

[3] V. V. Fedorchuk. Triples of infinite iterates of metrizable functors. *Mathematics of the USSR-Izvestiya*, 36(2):411–433, 1991.

[4] V. Gregori, S. Romaguera and A. Sapena, Uniform Continuity in Fuzzy Metric Spaces. *Rend. Istit. Mat. Univ. Trieste* Suppl. 2, Vol. XXXII : 81–88, 2001.

[5] A. George, P. Veeramani. On some results in fuzzy metric spaces. *Fuzzy Sets Syst.* 64 : 395–399, 1994.

[6] A. George, P. Veeramani. Some theorems in fuzzy metric spaces, *J. Fuzzy Math.* 3 : 933–940, 1995.

[7] V. Gregori, S. Romaguera. On completion of fuzzy metric spaces. *Fuzzy Sets and Systems.* 130 (3) : 399–404, 2002.

[8] J. Rodríguez-López, S. Romaguera. The Hausdorff fuzzy metric on compact sets. *Fuzzy Sets and Systems.* 147 (2) : 273–283, 2004.

[9] A. Savchenko. Fuzzy hyperspase monad. *Mat. Stud.* 33 : 192—198, 2010.

[10] O. Shukel'. Functors of finite degree and asymptotic dimension zero. *Mat. Stud.* 29 : 101–107, 2008.

[11] H. Torunczyk, J. West. A Hilbert spacelimit for the iterated hyperspace functor. *Proc. Amer. Math. Soc.* 89 : 329–335, 1983.

[12] M. Zarichnyi. Iterated superextensions, in: General topology, 1986, Moscow, Moscow University, 45–59, (in Russian).