

International
Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
(17.03.1928-18.12.2019)

May 25-28, 2021
Odesa, Ukraine

LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

SCIENTIFIC COMMITTEE

Drozd Yu.

(Kyiv, Ukraine)

Maksymenko S.

(Kyiv, Ukraine)

Plaksa S.

(Kyiv, Ukraine)

Prishlyak A.

(Kyiv, Ukraine)

Bakhtin O.

(Kyiv, Ukraine)

Balan V.

(Bucharest, Romania)

Banakh T.

(Lviv, Ukraine)

Borysenko O.

(Kharkiv, Ukraine)

Cherevko Ye.

(Odesa, Ukraine)

Fedchenko Yu.

(Odesa, Ukraine)

Karlova O.

(Chernivtsi, Ukraine)

Kiosak V.

(Odessa, Ukraine)

Konovenko N.

(Odessa, Ukraine)

Lyubashenko V.

(Kyiv, Ukraine)

Matsumoto K.

(Yamagata, Japan)

Mormul P.

(Warsaw, Poland)

Mykhailyuk V.

(Chernivtsi, Ukraine)

Plachta L.

(Krakov, Poland)

Pokas S.

(Odessa, Ukraine)

Sabitov I.

(Moscow, Russia)

Savchenko O.

(Kherson, Ukraine)

Sergeeva A.

(Odessa, Ukraine)

Shelekhov A.

(Tver, Russia)

Zarichnyi M.

(Lviv, Ukraine)

ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Lishchenko N. Dean of faculty of the computer systems and automation ONAFT

ORGANIZING COMMITTEE

Cherevko Ye.
Eftekharinasab K.
Fedchenko Yu.
Feshchenko B.
Khohlyk O.

Klishchuk B.
Konovenko N.
Kravchenko A.
Kuznietsova I.
Maksymenko S.

Osadchuk E.
Plakosh A.
Prus A.
Sergeeva A.
Soroka Yu.

Properties of quasisymmetric mappings to preserve the structures of spaces

Evgeniy Petrov

(Institute of Applied Mathematics and Mechanics of the NAS of Ukraine, Sloviansk, Ukraine)

E-mail: eugeniy.petrov@gmail.com

Ruslan Salimov

(Institute of Mathematics of the NAS of Ukraine, Kiev, Ukraine)

E-mail: ruslan.salimov1@gmail.com

The class of quasisymmetric mappings on the real axis was first introduced by A. Beurling and L. V. Ahlfors [1]. Later P. Tukia and J. Väisälä [2] considered these mappings between general metric spaces. See, e.g., [3] for an overview of the results in this direction. In our work we generalize the concept of quasisymmetric mappings to the case of general semimetric spaces. We establish conditions under which the image $f(X)$ of a semimetric space X with the triangle function Φ_1 under η -quasisymmetric embedding f is a semimetric space with another triangle function Φ_2 . Condition under which f preserves a Ptolemy inequality is also found as well as condition under which f preserves a relation “to lie between” imposed on three different points of the space.

Let X be a nonempty set. Recall that a mapping $d: X \times X \rightarrow \mathbb{R}^+$, $\mathbb{R}^+ = [0, \infty)$ is a *metric* if for all $x, y, z \in X$ the following axioms hold: (i) $(d(x, y) = 0) \Leftrightarrow (x = y)$, (ii) $d(x, y) = d(y, x)$, (iii) $d(x, y) \leq d(x, z) + d(z, y)$. The pair (X, d) is called a *metric space*. If only axioms (i) and (ii) hold then the pair (X, d) is called a *semimetric space*.

Definition 1. Let (X, d) , (Y, ρ) be semimetric spaces. We shall say that an embedding $f: X \rightarrow Y$ is η -quasisymmetric if there is a homeomorphism $\eta: [0, \infty) \rightarrow [0, \infty)$ so that

$$d(x, a) \leq td(x, b) \text{ implies } \rho(f(x), f(a)) \leq \eta(t)\rho(f(x), f(b))$$

for all triples a, b, x of points in X and for all $t > 0$.

A definition of a triangle function was introduced by M. Bessenyei and Z. Páles in [4].

Definition 2. Consider a semimetric space (X, d) . We say that $\Phi: \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a *triangle function* for d if Φ is symmetric and monotone increasing in both of its arguments, satisfies $\Phi(0, 0) = 0$ and, for all $x, y, z \in X$, the following generalized triangle inequality holds:

$$d(x, y) \leq \Phi(d(x, z), d(y, z)).$$

The most important triangle functions $\Phi(u, v)$ which generate well-known types of metrics and their generalizations are $u + v$ (metric), $K(u + v)$ (b -metric with $K \geq 1$), $\max\{u, v\}$ (ultrametric).

Proposition 3. Let (X, d) be a semimetric space with the triangle function Φ_1 , (Y, ρ) be a semimetric space and let $f: X \rightarrow Y$ be a surjective η -quasisymmetric embedding. Suppose that the following conditions hold for Φ_1 and for some function $\Phi_2: \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$:

- (i) Φ_2 is symmetric, monotone increasing in both of its arguments and satisfies $\Phi(0, 0) = 0$,
- (ii) $\lambda\Phi_1(x, y) \leq \Phi_1(\lambda x, \lambda y)$ and $\Phi_2(\lambda x, \lambda y) \leq \lambda\Phi_2(x, y)$ for every $\lambda > 0$,
- (iii) For every $t_1, t_2 \in \mathbb{R}_+ \setminus \{0\}$ the inequality

$$1 \leq \Phi_1\left(\frac{1}{t_1}, \frac{1}{t_2}\right) \text{ implies } 1 \leq \Phi_2\left(\frac{1}{\eta(t_1)}, \frac{1}{\eta(t_2)}\right). \quad (1)$$

Then Φ_2 is a triangle function for the space (Y, ρ) .

In what follows under Ptolemaic spaces we understand semimetric spaces (X, d) for which the well-known Ptolemy inequality

$$d(x, z)d(t, y) \leq d(x, y)d(t, z) + d(x, t)d(y, z)$$

holds. Note that this inequality does not imply the standard triangle inequality in (X, d) .

Proposition 4. *Let (X, d) be a Ptolemaic space, (Y, ρ) be a semimetric space and let $f: X \rightarrow Y$ be a surjective η -quasisymmetric embedding. If for every $t_1, t_2, t_3, t_4 \in \mathbb{R}^+$ the inequality*

$$t_1 t_2 t_3 t_4 \leq t_1 t_2 + t_3 t_4 \text{ implies } \eta(t_1)\eta(t_2)\eta(t_3)\eta(t_4) \leq \eta(t_1)\eta(t_2) + \eta(t_3)\eta(t_4), \quad (2)$$

then (Y, ρ) is also Ptolemaic.

Let (X, d) be a semimetric space and let x, y, z be different points from X . We shall say that the point y lies between x and z if the equality $d(x, z) = d(x, y) + d(y, z)$ holds. K. Menger [5] seems to be the first who formulated the concept of “metric betweenness” for general metric spaces.

Theorem 5. *Let (X, d) , (Y, ρ) be semimetric spaces and let $f: X \rightarrow Y$ be η -quasisymmetric embedding. If the homeomorphism η has the form*

$$\eta(t) = \begin{cases} \frac{1}{2} + \Psi_1(t, 1-t), & t \in [0, 1], \\ \frac{1}{\frac{1}{2} + \Psi_2(\frac{1}{t}, 1-\frac{1}{t})}, & t \in [1, \infty), \end{cases} \quad (3)$$

where Ψ_1, Ψ_2 are some continuous, antisymmetric, strictly increasing by the first variables, defined on $[0, 1] \times [0, 1]$ functions of two variables such that $\Psi_1(1, 0) = \Psi_2(1, 0) = 1/2$, then f preserves metric betweenness.

REFERENCES

- [1] Arne Beurling, Lars V. Ahlfors. The boundary correspondence under quasiconformal mappings. *Acta Mathematica*, 96:125–142, 1956.
- [2] Pekka Tukia, Jussi Väisälä. Quasisymmetric embeddings of metric spaces. *Annales Academiae Scientiarum Fennicae. Series A I. Mathematica*, 5:97–114, 1980.
- [3] Juha Heinonen. *Lectures on analysis on metric spaces*. New York: Springer, 2001.
- [4] Mihály Bessenyei, Zsolt Páles. A contraction principle in semimetric spaces. *Journal of Nonlinear and Convex Analysis*, 18(3):515–524, 2017.
- [5] Karl Menger. Untersuchungen über allgemeine Metrik. *Mathematische Annalen*, 100:75–163, 1928.

Y. Chapovskyi, D. Efimov, A. Petravchuk <i>Centralizers of elements in Lie algebras of vector fields</i>	113
E. Petrov, R. Salimov <i>Properties of quasisymmetric mappings to preserve the structures of spaces</i>	114
D. R. Popovych <i>IW contractions and their generalizations</i>	116
Prabhjot Singh <i>Weak Separation Condition coincides Finite Type Condition</i>	118
Pratyush Pranav <i>Topological data analysis for cosmology: theory and applications</i>	120
A. Prishlyak, Ch. Hatamian <i>Morse flows with singularities on boundary of 3-manifolds</i>	121
V. Prokip <i>On the matrix equation $AX - YB = C$ over Bezout domains</i>	122
A. Prykarpatsky, I. Mykytyuk <i>On the metric equations generated by symplectic deformations on $P_2(C)$</i>	124
O. Reinov <i>On a result of G. Pisier concerning Sidon sets</i>	126
A. K. Sadullaev, F. G. Mukhamadiev <i>On the τ-placedness of space of the permutation degree</i>	127
T. N Safarov <i>The analogue of Darboux equation in Galilean space</i>	128
V. M. Safonov, I. V. Zamrii, O. V. Safonova <i>On countable multiplicity of mappings</i>	129
A. Savchenko <i>Fuzzy ultrametrization of spaces of non-additive measures on fuzzy ultrametric spaces</i>	130
O. Sazonova <i>About one class of continual approximate solutions with arbitrary density</i>	131
J. Segert <i>Painlevé VI Solutions From Equivariant ADHM Instanton Bundles</i>	133
A. Serdyuk, I. Sokolenko <i>Asymptotic estimates for the widths of classes of periodic functions of high smoothness</i>	135
H. Sinyukova <i>Some generalizations of the known theorems of the type of geodesical unique definability</i>	138
S. Som, A. Bera, L. K. Dey <i>Some remarks on the Metrizable of F-metric spaces</i>	139
P. Stegantseva, M. Grechneva <i>The surfaces with the flat normal connection and the constant curvature of Grassmann image in Minkowski space</i>	140
A. Skryabina, P. Stegantseva <i>The relation between T_0-topologies with the weight $2^{n-2} < k \leq 2^{n-1}$ on n-element set and T_0-topologies close to the discrete on $(n - 1)$-element set</i>	142
Ya. B. Stelmakh <i>The Golomb and Kirch topologies on the set of nonzero integers</i>	143
D. Dmitrishin, A. Stokolos <i>On symmetrization of univalent polynomials</i>	144