



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

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Odesa, Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Definition 1. Let E be a Banach lattice. An operator $T : E \rightarrow E$ is said to be an unbounded demi KB operator if, for every positive increasing sequence (x_n) in the closed unit ball of E such that $(x_n - Tx_n)$ is unbounded norm convergent to $x \in E$, there is an unbounded norm convergent subsequence of (x_n) .

Theorem 2. Let E be a Banach lattice. Every KB operator $T : E \rightarrow E$ is unbounded demi KB operator.

In this study, we characterize the operators on Banach lattices that under which conditions they satisfy unbounded demi KB operators.

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On polynomial and regular maps of spheres

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This talk offers some results on to the intersection of algebraic topology and algebraic geometry.

Let K be a field and $X \subseteq K^m$, $Y \subseteq K^n$ algebraic sets. Recall that a map $f = (f_1, \dots, f_n) : X \rightarrow Y$ is called *polynomial* (resp. *regular*) if there are polynomials $F_i, G_i \in \mathbb{R}[X_1, \dots, X_m]$ such that $f_i(x) = F_i(x)$ (resp. $f_i(x) = \frac{F_i(x)}{G_i(x)}$, $G_i(x) \neq 0$) with $i = 1, \dots, n$ for $x \in X$.

Remark 1. If K is a algebraically closed field then the only regular maps of algebraic sets are polynomial maps.

Example 2. (1) Let $K = \mathbb{R}$ or \mathbb{C} , the fields of reals or complex numbers. The n -sphere

$$\mathbb{S}^n(K) = \{(x_1, \dots, x_{n+1}) \in \mathbb{K}^{n+1}; x_1^2 + \dots + x_{n+1}^2 = 1\} = V(X_0^2 + \dots + X_n^2 - 1)$$

is an algebraic set in \mathbb{K}^{n+1} . Write $\mathbb{S}^n(\mathbb{R}) = \mathbb{S}^n$ and notice a diffeomorphism $\mathbb{S}^n(\mathbb{C}) \approx T\mathbb{S}^n$, the tangent bundle of \mathbb{S}^n . Consequently, a homotopy equivalence $\mathbb{S}^n(\mathbb{C}) \simeq \mathbb{S}^n$.

(2) Let $K = \mathbb{R}, \mathbb{C}, \mathbb{H}$ with the skew \mathbb{R} -algebra \mathbb{H} of quaternions. The Grassmannian (of r -planes in K^n), can be identified with $G_{n,r}(K) = \{A \in M_n(K); A^2 = A, \bar{A} = A^t, \text{rk}(A) = r\}$ for the set $M_n(K)$ of all $n \times n$ -matrices over K .

But, for any idempotent $n \times n$ matrix over K , its rank coincides with the trace. Therefore, $G_{n,r}(K)$ can be viewed as a real affine variety.

Let $X \subseteq \mathbb{R}^m$, $Y \subseteq \mathbb{R}^n$ be algebraic sets. Write $[X, Y]$ for the set of homotopy classes of continuous maps and $[X, Y]_{alg}$ the subset of $[X, Y]$ represented by regular maps. One of the main purposes of the talk is to estimate the size of $\pi_m(\mathbb{S}^n)_{alg} = [\mathbb{S}^m, \mathbb{S}^n]_{alg}$ in $\pi_m(\mathbb{S}^n) = [\mathbb{S}^m, \mathbb{S}^n]$.

Basing on [1], [3] and [4], we aim to show:

Theorem 3. *If $k = 0, 1, \dots, 7$ then elements of $\pi_{n+k}(\mathbb{S}^n)$ can be represented by regular maps for $n \geq 1$.*

Next, we make use of [2] to show a homeomorphism $TG_{n,r}(K) \xrightarrow{\cong} \text{Idem}_{n,r}(K)$ for the tangent bundle $TG_{n,r}(K)$ of $G_{n,r}(K)$ and $\text{Idem}_{n,r}(K)$, the set of all idempotent $n \times n$ matrices with rank r for $K = \mathbb{R}, \mathbb{C}, \mathbb{H}$. Finally, we present:

Theorem 4. *If $K = \mathbb{R}, \mathbb{C}, \mathbb{H}$ then there is:*

- (1) *a regular deformation retraction $\text{Idem}_{n,r}(K) \rightarrow G_{n,r}(K)$;*
- (2) *an injection $\mathcal{P}_{\mathbb{C}}[V_{\mathbb{C}}, \text{Idem}_{n,r}(K)] \rightarrow \mathcal{R}_{\mathbb{R}}[V, G_{n,r}(K)]$ from the sets of homotopy classes of complex-valued polynomial to such a set of real-valued regular maps, where $V_{\mathbb{C}}$ denotes the Zariski closure in the affine space \mathbb{C}^n of a subset $V \subseteq \mathbb{R}^n$.*

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On homomorphisms of bicyclic extensions of archimedean totally ordered groups

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We follow the terminology of [1, 2]. Let G^+ be the positive cone of a totally ordered group. On the set $\mathcal{B}^+(G) = G^+ \times G^+$ we define the semigroup operation “ \cdot ” in the following way

$$(a, b) \cdot (c, d) = \begin{cases} (c \cdot b^{-1} \cdot a, d), & \text{if } b < c; \\ (a, d), & \text{if } b = c; \\ (a, b \cdot c^{-1} \cdot d), & \text{if } b > c, \end{cases}$$

for $a, b, c, d \in G^+$.

Theorem 1. *Let G and H be archimedean totally ordered groups. Then every o -homomorphism $\hat{\varphi}: G \rightarrow H$ generates a monoid homomorphism $\tilde{\varphi}: \mathcal{B}^+(G) \rightarrow \mathcal{B}^+(H)$, and every monoid homomorphism $\tilde{\varphi}: \mathcal{B}^+(G) \rightarrow \mathcal{B}^+(H)$ generates an o -homomorphism $\hat{\varphi}: G \rightarrow H$, which agree*

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