International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of

Dvytro Grave

(25.08.1863 - 19.12.1939)

Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023

Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Theorem 2. *Let $(X, \mathcal{A})$ be a Backström non-commutative curve, $(X, \tilde{\mathcal{A}})$ be its Auslander envelope.*

(1) gl.dim $\tilde{\mathcal{A}} \leq 2$.
(2) der.dim $\mathcal{A} \leq 2$, *where* der.dim $\mathcal{A}$ *denotes the* derived dimension *of* $\mathcal{A}$, *that is the Rouquier dimension* [2] *of the perfect derived category* $\mathcal{D}^{perf}(\mathrm{Coh}\,\mathcal{A})$.

Local versions of these results are proved in [1].

We also study the action of finite groups on Backström curves and prove the following theorem.

Theorem 3. *Let a finite group of order n acts on a Backström curve $(X, \mathcal{A})$ and* $\mathrm{char}\,\Bbbk \nmid n$. *Then the crossed product $(X, \mathcal{A} * G)$ is also a Backström curve and its Auslander envelope is $(X, \tilde{\mathcal{A}} * G)$.*

Some examples will also be presented.

References

[1] Yuriy Drozd. Backström algebras. arXiv: 2206.12875 [mathRT], 2022.
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On geodesic lines of Riemannian metric for Navier-Stokes equations

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Theorem 1. *The 14D Riemann metric in local coordinates*

$$\vec{x} = (x, y, z, t, \eta, \rho, m, u, v, w, p, \xi, \chi, n)$$

$$\begin{aligned} ds^2 = {} & 2\,dxdu + 2\,dydv + 2\,dzdw + (-W(\vec{x},t)w - V(\vec{x},t)v - U(\vec{x},t)u)\,dt^2 + \\ & + \left(-U(\vec{x},t)p - u\,(U(\vec{x},t))^2 - uP(\vec{x},t) + w\mu\,\frac{\partial}{\partial z}U(\vec{x},t) - wU(\vec{x},t)W(\vec{x},t)\right) d\eta^2 + \\ & + \left(v\mu\,\frac{\partial}{\partial y}U(\vec{x},t) - vU(\vec{x},t)V(\vec{x},t) + u\mu\,\frac{\partial}{\partial x}U(\vec{x},t)\right) d\eta^2 + 2\,d\eta d\xi + 2\,d\rho d\chi + 2\,dmdn + \\ & + \left(-V(\vec{x},t)p - vP(\vec{x},t) - v\,(\vec{x},t))^2 - V(\vec{x},t)W(\vec{x},t)w + v\mu\,\frac{\partial}{\partial y}V(\vec{x},t) - uU(\vec{x},t)V(\vec{x},t)\right) d\rho^2 + \\ & + \left(u\mu\,\frac{\partial}{\partial x}V(\vec{x},t)\right) d\rho^2 + \left(-uU(\vec{x},t)W(\vec{x},t) - w\,(W(\vec{x},t))^2 - wP(\vec{x},t) + w\mu\,\frac{\partial}{\partial z}W(\vec{x},t)\right) dm^2 + \\ & + \left(v\mu\,\frac{\partial}{\partial y}W(\vec{x},t) - vV(\vec{x},t)W(\vec{x},t) + u\mu\,\frac{\partial}{\partial x}W(\vec{x},t) - W(\vec{x},t)p\right) dm^2 \end{aligned} \quad (1)$$

is the Ricci-flat,

$$R_{44} = U_x + V_y + W_z = 0,\ R_{55} = 0,\ R_{66} = 0,\ R_{77} = 0$$

on solutions of Navier-Stokes system of equations

$$\frac{\partial}{\partial t}\vec{Q}(\vec{x},t) + (\vec{Q}(\vec{x},t)\cdot\vec{\nabla})\vec{Q}(\vec{x},t) - \mu\,\Delta\vec{Q}(\vec{x},t) + \vec{\nabla}P(\vec{x},t) = 0, \quad \vec{\nabla}\cdot\vec{Q}(\vec{x},t) = 0, \qquad (2)$$

where $\vec{Q}(\vec{x},t) = [U(\vec{x},t), V(\vec{x},t), W(\vec{x},t)]$ *are the components of velocity and* $P(\vec{x},t)$ *is pressure of liquid.(see e.g. [1-2])*

To obtain the metric (1) presentation the NS-system of equations in the form of laws conservations

$$U_t + (U^2 - \mu U_x + P)_x + (UV - \mu U_y)_y + (UW - \mu U_z)_z = 0$$
$$V_t + (V^2 - \mu V_y + P)_y + (UV - \mu V_x)_x + (VW - \mu V_z)_z = 0,$$
$$W_t + (W^2 - \mu W_z + P)_z + (UW - \mu W_x)_x + (VW - \mu W_y)_y = 0,$$
$$(U_x + V_y + W_z) = 0,$$

is used.

The metric (1) belongs to the class of the Riemann spaces with vanishing scalar Invariants. Their geodesics with respect to the coordinates $\eta, \rho, m, \xi, \chi, n$ has form of equations direct lines

$$\ddot{\eta} = 0,\ \ddot{\rho} = 0,\ \ddot{m} = 0,\ \ddot{\xi} = 0,\ \ddot{\chi} = 0,\ \ddot{n} = 0,$$

and in this sense to them the partially-projective spaces of V.Kagan corresponds.

For the coordinates $[x, y, z, t]$ the equations of geodesics of metric (1) are

$$\begin{aligned}\frac{d^2}{ds^2}x(s) &= 1/2\,(\dot{m}(s))^2\,U(x,y,z,t)\,W(x,y,z,t) - 1/2\,(\dot{m}(s))^2\,\mu\,\frac{\partial}{\partial x}W(x,y,z,t) + \\ &+ 1/2\,(\dot{\eta}(s))^2\,(U(x,y,z,t))^2 + 1/2\,(\dot{\rho})^2\,U(x,y,z,t)\,V(x,y,z,t) - \\ &- 1/2\,(\dot{\rho})^2\,\mu\,\frac{\partial}{\partial x}V(x,y,z,t) - -1/2\,(\dot{\eta})^2\,\mu\,\frac{\partial}{\partial x}U(x,y,z,t) + \\ &+ 1/2\,U(x,y,z,t)\left(\frac{d}{ds}t(s)\right)^2 + 1/2\,(\dot{\eta}(s))^2\,P(x,y,z,t),\\ \frac{d^2}{ds^2}t(s) &= 1/2\,W(x,y,z,t)\left(\frac{d}{ds}m(s)\right)^2 + 1/2\,U(x,y,z,t)\left(\frac{d}{ds}\eta(s)\right)^2 + \\ &\quad + 1/2\,V(x,y,z,t)\left(\frac{d}{ds}\rho(s)\right)^2,\\ \frac{d^2}{ds^2}y(s) &= \cdots,\\ \frac{d^2}{ds^2}z(s) &= \cdots.\end{aligned}$$

The equations of geodesics for dual coordinates $[u, v, w, p]$ form the linear system of the second order equations

$$\frac{d^2}{ds^2}u(s) = A_1\,u(s) + B_1\,v(s) + C_1\,w(s) + E_1\,p(s),$$

$$\frac{d^2}{ds^2}v(s) = A_2 u(s) + B_2 v(s) + C_2 w(s) + E_2 p(s),$$
$$\frac{d^2}{ds^2}w(s) = A_3 w(s) + B_3 v(s) + C_3 w(s) + E_3 p(s),$$
$$\frac{d^2}{ds^2}p(s) = A_4 u(s) + B_4 v(s) + C_4 w(s) + E_4 p(s),$$

with the coefficients depending on the solutions $U(x,y,z,t), V(x,y,z,t), W(x,y,z,t), P(x,y,z,t$ of the system (2).

On the base of solutions of equations for the Killing vectors of the metric

$$K_{i,j} + K_{j,i} - 2\Gamma^k_{ij}K_k = 0, \quad or \quad K^k g_{ij,k} + g_{ik}K^k, j + g_{jk}K^k, i = 0, \tag{4}$$

a new examples of reductions and solutions of the system (2) are constructed.

Properties of the Lie derivative for the connection coefficients of the metric (1) and the vector field of the form $u^i = g^i_k v^k$

$$u^i_{j,k} + u^n\Gamma^i_{jk},_n + u^n_{,j}\Gamma^i_{nk} + u^n_{,k}\Gamma^i_{jn} - u^n_{,n}\Gamma^i_{jk} = 0,$$

where Γ^i_{jk}-are the coefficients of connection of the metric (1) with the aim of constructing new examples of solutions to the system (2) are discussed.

Another possibility for studying the properties of the NS system by the geometric method is the use of differential Beltrami parameters of the metric (1) $\Delta_2(f) = g^{ij}\frac{\partial^2 f}{\partial x_i, x_j} - \Gamma^k_{ij}\frac{\partial f}{\partial x_k}$. As example, in particular case $f = \psi(x,y,z,t,0,0,0,u,v,w,p,0,0,0)$, from solutions of the linear equation with variable coefficients $\Delta_2(f) = 0$ the relation

$$(U(\vec{x},t) - W(\vec{x},t))P(\vec{x},t)/\mu = U(\vec{x},t)\frac{\partial}{\partial z}U(\vec{x},t) - W(\vec{x},t)\frac{\partial}{\partial x}V(\vec{x},t) -$$
$$- W(\vec{x},t)\frac{\partial}{\partial x}U(\vec{x},t) - W(\vec{x},t)\frac{\partial}{\partial x}W(\vec{x},t) + \frac{\partial}{\partial z}V(\vec{x},t)U(\vec{(}x,t) + \frac{\partial}{\partial z}W(\vec{x},t)U(\vec{x},t),$$

between velocity and pressure can be derived and that can be applied to the studying properties of solutions of the system (2).

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