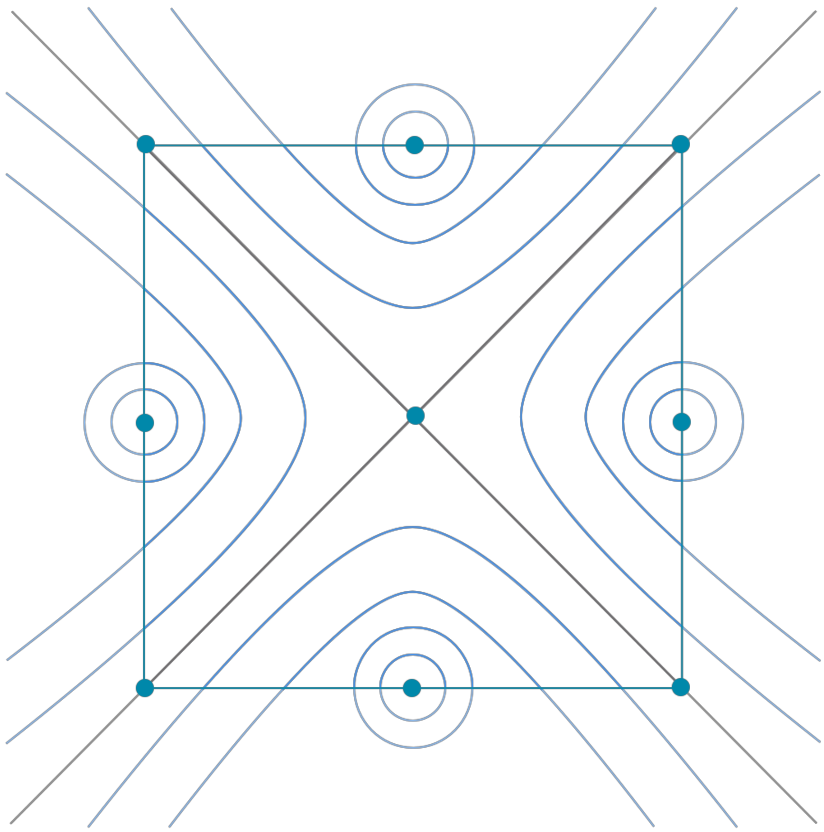
International Conference

Morse theory and its applications



dedicated to the memory and 70th anniversary
of Volodymyr Sharko (25.09.1949 — 07.10.2014)

Institute of Mathematics of National Academy of
Sciences of Ukraine, Kyiv, Ukraine

September 25—28, 2019

The conference is dedicated to the memory and 70th anniversary of the outstanding topologist, Corresponding Member of National Academy of Sciences **Volodymyr Vasylyovych Sharko** (25.09.1949-07.10.2014) to commemorate his contributions to the Morse theory, K-theory, L^2-theory, homological algebra, low-dimensional topology and dynamical systems.

List of topics

- Morse theory
- Low dimensional topology
- K-theory
- L^2-theory
- Dynamical systems
- Geometric methods of analysis

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Projective invariants of rational mappings

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We consider the k-jet spaces $\mathbf{J}^k$ as k-jets of analytical mappings $\mathbb{C}\boldsymbol{P}^1 \to \mathbb{C}\boldsymbol{P}^1$ equipped with adjoint $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$-action:

$$f \mapsto A \circ f \circ A^{-1},$$

where $A \in \mathbb{P}\mathbf{SL}_2(\mathbb{C})$ (cf. [3]).

The representations of corresponding Lie algebras by vector fields on $\mathbf{J}^0$ is the following:

$$\langle \partial_z + \partial_u, z\partial_z + u\partial_u, z^2\partial_z + u^2\partial_u \rangle.$$

Theorem 1. (1) *The field of adjoint invariants* [1], [2], *is generated by differential invariants of the second and third orders*

$$J_2 = u_1^{-3} \left((z-u)\, u_2 + 2u_1 \left(u_1 + 1\right)\right)^2,$$
$$J_3 = (z-u)^2 u_1^{-2} u_3 + 6 \left(z-u\right) u_1^{-2} \left(u_1+1\right) u_2 + 6 \left(u_1 + u_1^{-1}\right),$$

and invariant derivation

$$\nabla = \frac{(z-u)\, u_1 u_2 + 2u_1^2 \left(u_1+1\right)}{(z-u)\left(2u_1u_3 - 3u_2^2\right)} \frac{d}{dz}.$$

(2) *This field separates regular orbits.*

Equation $J_2\left(f\right) = 0$ has solutions of the form:

$$f\left(z\right) = \frac{ax+b}{cx+d}$$

where matrix

$$A = \begin{Vmatrix} a & b \\ c & d \end{Vmatrix}$$

has zero trace.

In general, values of invariant J_2 on linear fractional maps is equal to

$$J_2\left(f\right) = 4\frac{\left(trA\right)^2}{\det A},$$

and

$$J_3\left(f\right) = 6\frac{trA^2}{\det A} + 24.$$

We say that a rational mapping $f\left(z\right)$ is *singular* if $f\left(z\right)$ is linear fractional.

There exists an irreducible polynomial (we call it *generating polynomial*)

$$P_f\left(X, Y\right) \in \mathbb{C}[X, Y]$$

such that $P_f\left(J_2\left(f\right), J_3\left(f\right)\right) = 0$ or in other words, the mapping f, as well as all mappings that are $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ - equivalent to f are solutions of the third order differential equation

$$P_f\left(J_2, J_3\right) = 0.$$

Theorem 2. (1) *Two regular rational mappings f and g are $\mathbb{P}\mathbf{SL}_2(\mathbb{C})$ - equivalent if and only if their adjoint generating polynomial are proportional.*

(2) *Orbit of a regular rational mapping f consists of the solution space of ordinary differential equation*

$$P_f\left(J_2, J_3\right) = 0.$$

(1) For linear fractional transformations $f\left(z\right) = \frac{az+b}{cz+d}$, $ad - bc = 1$ we have

$$J_2(f) = 4(c+d)^2$$

and therefore the corresponding differential equation has the following form:

$$u_1^{-3}\left(\left(z-u\right)u_2 + 2u_1\left(u_1+1\right)\right)^2 - 4(a+d)^2 = 0$$

with 2-dimensional space of solutions.

(2) For the Jukowski mapping $f\left(z\right) = \frac{z^2+1}{2z}$ we have

$$P_f\left(X, Y\right) = -4X + 3Y - 45$$

and therefore the corresponding differential equation has the following form

$$\left(z-u\right)^2\left(3u_1u_3 - 4u_2^2\right) + 2\left(z-u\right)u_1\left(u_1+1\right)u_2 +$$

$$+ u_1^2\left(2u_1 - 1\right)\left(u_1 - 2\right) = 0.$$

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