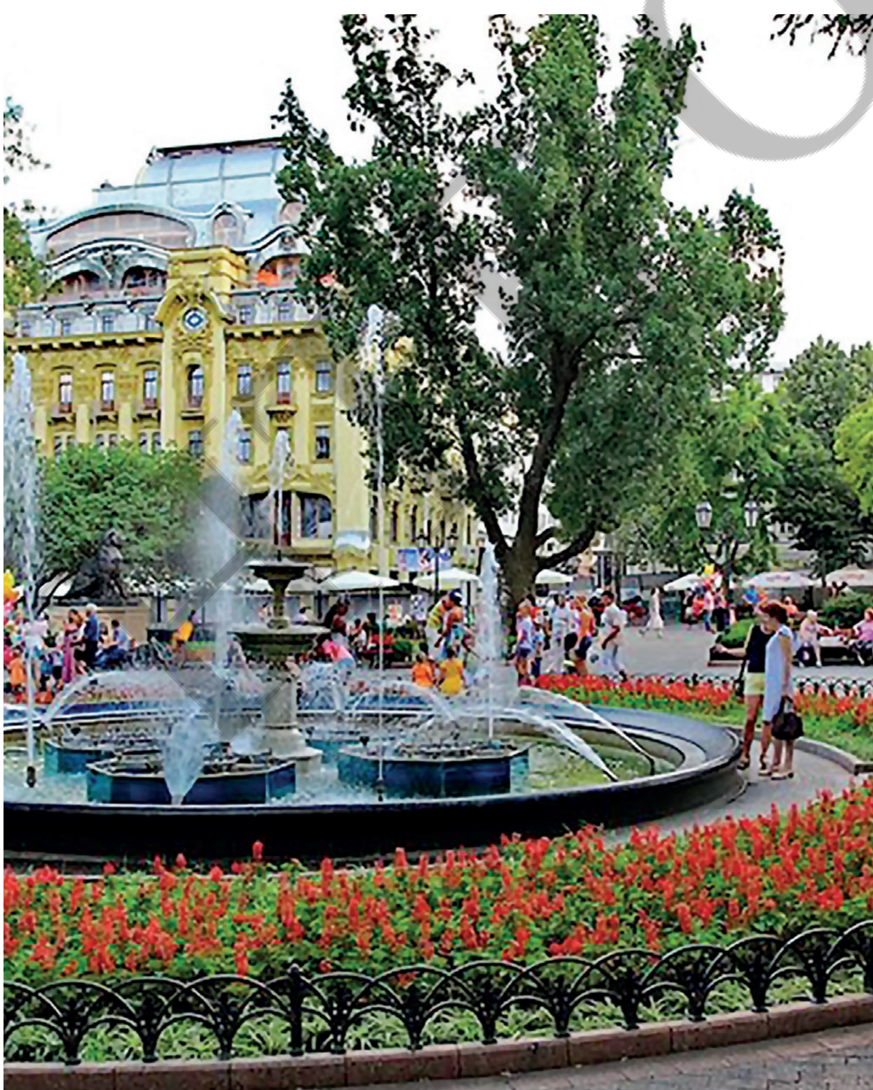
International
Scientific Conference

# Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of

## Dvytro Grave

(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Proximal connectedness. Spatially and descriptively connected spaces


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In this paper, we give results for spatially-connected spaces $(X, \delta)$ [5] (a widely-considered proximity space) and descriptively-connected spaces $(X, \delta_\Phi)$, a recent form of proximity space [3] with a number of applications. Definition 1 is analogous to connectedness in digital topology [2].

**Definition 1.** Let $(X, \delta)$ be a proximity space. Then two nonempty subsets $A$ and $B$ are $\delta$-connected, provided there exists a finite family of subsets $\{E_i\}_{i=0}^n$ of $X$ such that $A = E_0$, $B = E_n$, and $E_i \ \delta \ E_{i+1}$ for all $i = 0, 1, \ldots, n-1$. A proximity space $(X, \delta)$ is said to be connected, provided any pair of subsets $A, B$ of $X$ are $\delta$-connected.

**Example 2.** In Figure 1.1, let $h, k : I \to K$ be continuous maps called homotopies in a space $K$. A homotopic class for different maps $h$ (denoted by $[h] = \left\{h_0, \ldots, h_i, \ldots, h_{n-1[n]}\right\}$ with $[n] = \bmod n \in \mathbb{Z}^+$) is a collection of $h_{i[n]}$ homotopic maps that have the same endpoints, namely, $h_i(0)$ and $h_i(1)$. The maps in $[h]$ are spatially near. Similarly, the maps in $[k]$ are spatially near. However, the homotopy classes $[h], [k]$ are spatially far. Also, from Definition 1,

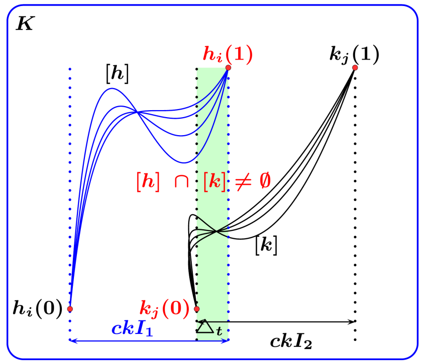


FIGURE 1.1. Spatially near homotopy maps in spatial far homotopy classes $[h], [k]$

every pair of nonempty subsets $A, B \in [h]$ are $\delta$-connected, since $A \ \delta \ B$. Likewise, the maps in $[k]$ are $\delta$-connected.

**Theorem 3.** *Let $(X, \delta_1)$ and $(Y, \delta_2)$ be proximity spaces. Then a map $f : X \to Y$ is proximally continuous if and only if a pair of $\delta_1$-connected subsets of $X$ is mapped to a pair of $\delta_2$-connected subsets of $Y$.*

**Definition 4.** [1] Let $(X, \delta_\Phi)$ be a descriptive proximity space. Then two subsets $A$ and $B$ are $\delta_\Phi$**-connected**, provided there exists a finite family of subsets $\{D_i\}_{i=0}^{n}$ of $X$ such that $D_0 = A$, $D_n = B$, and $D_i \ \delta_\Phi \ D_{i+1}$ for all $i = 0, 1, \ldots, n-1$. A descriptive proximity space $(X, \delta_\Phi)$ is said to be **descriptively connected**, provided any pair of subsets $A, B$ of $X$ are $\delta_\Phi$-connected.

**Proposition 5.** *In a digital topology space $X$, descriptively near sets in a digital image $X$ are descriptively connected.*

**Theorem 6.** *Let $(X, \delta_{\Phi_1})$ and $(Y, \delta_{\Phi_2})$ be descriptive proximity spaces. Then a map $f : X \to Y$ is descriptive proximally continuous if and only a pair of $\delta_{\Phi_1}$-connected subsets of $X$ is mapped to a pair of $\delta_{\Phi_2}$-connected subsets of $Y$.*

Given a set $X \subset \mathbb{Q} \times \mathbb{Q}$, a digital image on $X$ is a map $Img : X \to \mathbb{R}$ so that each picture element $p \in X$ aka (sub)pixels or (sub)voxels has location $\mathbb{Q} \times \mathbb{Q}$ and value $Img(p) \in \mathbb{R}$. There are two distinct type digital images, namely, frames (denoted by $Img_t$ or simply by $fr_t$) which is a time-ordered sequence of images in which each frame occurs at an elapsed time $t$ in a video, and single images (denoted by $Img$). The intersection of the closures of bounded regions with nonempty interiors give rise to $\underset{\kappa}{\delta}$-connectedness.

**Definition 7.** Given a digital image $Img$, two subimages $A$ and $B$, are adjacent, denoted by $A \underset{\kappa}{\delta} B$, provided there exist pixels $p \in A$ and $q \in B$ such that $p = q$ or $p$ and $q$ are adjacent.

**Definition 8.** A bounded region $E \subset fr$ with a non-empty interior is said to be $\underset{\kappa}{\delta}$-connected, provided for each pair of distinct voxels $p$ and $q$ in $E$, there exists a finite sequence of voxels $p = v_0, v_1, \ldots, v_n = q$ such that each pair of consecutive voxels $v_i$ and $v_{i+1}$ is $\underset{\kappa}{\delta}$-connected for all $i = 0, 1, \ldots, n-1$.

**Proposition 9.** *$\underset{\kappa}{\delta}$-connected regions are preserved under a continuous digital functions.*

**Example 10.** Consider a moving object appears as a bounded foreground region recorded in a video frame $fr_t$. Observe that the moving object -as a bounded region- is partitioned into (bounded) $\underset{\kappa}{\delta}$ connected subregions, which cover the moving object with a minimum number of contractible subregions is a form of geometric Lusternik-Schirel'mann category [4].

Spatially near subsets in a digital image (*i.e.*, subsets that share points) reside in a discrete proximity space.

**Definition 11.** For any pair of nonempty subsets $A, B$ in a digital image $Img$, $A$ is near $B$ (denoted by $A\ \delta\ B$), provided $A$ and $B$ have points in common, *i.e.*, $A\ \delta\ B$ iff $A \cap B \neq \emptyset$. Hence, $\delta$ is a discrete proximity relation and $(Img, \delta)$ is a discrete proximity space [6, §40.2, pp. 266-267].

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# $p$-Hyperbolic Zolotarev functions in boundary value problems for a $p$ th order differential operator


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For the self-adjoint operator of the $p$th derivative, a system of fundamental solutions is constructed. This system is analogues to the classical system of sines and cosines. The properties of such functions are studied. Classes of self-adjoint boundary conditions are described. For the operator of the third derivative, the resolvent is calculated and an orthonormal basis of eigenfunctions is given.