



International  
Scientific Conference



# Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of  
**Dvytro Grave**  
(25.08.1863 - 19.12.1939)  
Academician of the Ukrainian  
Academy of Sciences, the  
first director of the Institute of  
Mathematics of NAS of Ukraine

May 29 – June 1, 2023  
Odesa, Ukraine

## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

## ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

## SCIENTIFIC COMMITTEE

- |  |   |
|--|---|
| • <b>Bolotov D.</b> ( <i>Kharkiv, Ukraine</i> )  | • <b>Konovenko N.</b> ( <i>Odesa, Ukraine</i> )   |
| • <b>Bondarenko V.</b> ( <i>Kyiv, Ukraine</i> )  | • <b>Maksymenko S.</b> ( <i>Kyiv, Ukraine</i> )   |
| • <b>Boychuk O.</b> ( <i>Kyiv, Ukraine</i> )     | • <b>Mikhailets V.</b> ( <i>Kyiv, Ukraine</i> )   |
| • <b>Boyko V.</b> ( <i>Kyiv, Ukraine</i> )       | • <b>Ostrovskiy V.</b> ( <i>Kyiv, Ukraine</i> )   |
| • <b>Cherevko Ye.</b> ( <i>Odesa, Ukraine</i> )  | • <b>Petravchuk A.</b> ( <i>Kyiv, Ukraine</i> )   |
| • <b>Dorogovtsev A.</b> ( <i>Kyiv, Ukraine</i> ) | • <b>Plaksa S.</b> ( <i>Kyiv, Ukraine</i> )       |
| • <b>Drozd Yu.</b> ( <i>Kyiv, Ukraine</i> )      | • <b>Portenko M.</b> ( <i>Kyiv, Ukraine</i> )     |
| • <b>Gerasymenko V.</b> ( <i>Kyiv, Ukraine</i> ) | • <b>Pratsiovytyi M.</b> ( <i>Kyiv, Ukraine</i> ) |
| • <b>Fedchenko Yu.</b> ( <i>Odesa, Ukraine</i> ) | • <b>Savchenko O.</b> ( <i>Kherson, Ukraine</i> ) |
| • <b>Kiosak V.</b> ( <i>Odesa, Ukraine</i> )     | • <b>Romanyuk A.</b> ( <i>Kyiv, Ukraine</i> )     |
| • <b>Kochubei A.</b> ( <i>Kyiv, Ukraine</i> )    | • <b>Timokha O.</b> ( <i>Kyiv, Ukraine</i> )      |

## ORGANIZING COMMITTEE

- |  |   |
|--|---|
| • <b>Maksymenko S.</b> ( <i>Kyiv, Ukraine</i> )  | • <b>Cherevko Ye.</b> ( <i>Odesa, Ukraine</i> ) |
| • <b>Konovenko N.</b> ( <i>Odesa, Ukraine</i> )  | • <b>Osadchuk Ye.</b> ( <i>Odesa, Ukraine</i> ) |
| • <b>Fedchenko Yu.</b> ( <i>Odesa, Ukraine</i> ) | • <b>Sergeeva O.</b> ( <i>Odesa, Ukraine</i> )  |

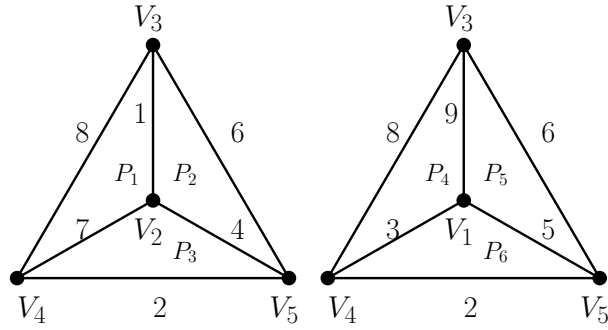


FIGURE 2.3. Degree 6 non-planar degeneration

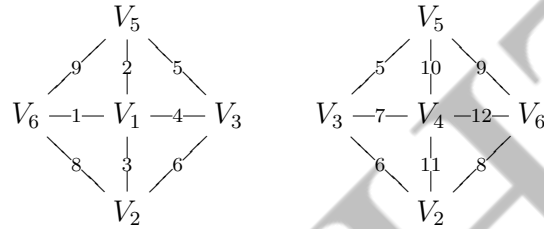


FIGURE 2.4. Degree 8 non-planar degeneration

## REFERENCES

- [1] M. Amram, C. Gong, J.-L. Mo, "On the Galois covers of degenerations of surfaces of minimal degree", *Mathematische Nachrichten*, 2023. <https://doi.org/10.1002/mana.202100183>.
- [2] M. Amram, "Fundamental groups of Galois covers as tools to study non-planar degenerations", <https://arxiv.org/abs/2104.02781>.
- [3] M. Amram, C. Gong, P. Kumar Roy, U. Sinichkin, U. Vishne, "The fundamental group of Galois covers of surfaces of degree 8", <https://doi.org/10.48550/arXiv.2303.05241>

## Surfaces with zero mean curvature vector in 4-dimensional spaces

Naoya Ando

(Faculty of Advanced Science and Technology, Kumamoto University, 2-39-1 Kurokami,  
Kumamoto 860-8555 Japan)

*E-mail:* andonaoya@kumamoto-u.ac.jp

Let  $(N, h)$  be an oriented Riemannian 4-manifold. Let  $\wedge^2 TN$  be the 2-fold exterior power of the tangent bundle  $TN$  of  $N$ . Then  $\wedge^2 TN$  is a vector bundle of rank 6 over  $N$  and Hodge's  $*$ -operator gives a bundle decomposition  $\wedge^2 TN = \wedge^2_+ TN \oplus \wedge^2_- TN$  by two subbundles  $\wedge^2_{\pm} TN$  of rank 3. The twistor spaces associated with  $N$  are the sphere bundles in  $\wedge^2_{\pm} TN$  and denoted by  $U(\wedge^2_{\pm} TN)$ . We can refer to [5] for twistor spaces. Let  $M$  be a Riemann surface and  $F : M \rightarrow N$  a conformal and minimal immersion. Let  $F^*TN$  be the pull-back bundle on  $M$  by  $F$ . Then  $F$  gives its twistor lifts, which are sections of  $U(\wedge^2_{\pm} F^*TN)$ . Let

$\sigma$  be the second fundamental form of  $F$ . Let  $w$  be a local complex coordinate of  $M$  and set  $\sigma_{ww} := \sigma(\partial/\partial w, \partial/\partial w)$ . Then  $h(\sigma_{ww}, \sigma_{ww})dw^4$  does not depend on the choice of  $w$  and therefore  $F$  gives a complex quartic differential  $Q$  on  $M$ . If  $N$  is a 4-dimensional space form, then  $Q$  is holomorphic. Isotropy of  $F$  is given by  $Q \equiv 0$  and this condition is equivalent to horizontality of a twistor lift of  $F$  ([6], [4]).

Let  $(N, h)$  be an oriented neutral 4-manifold. Then the metric  $\hat{h}$  of  $\Lambda^2 TN$  induced by  $h$  has signature  $(2, 4)$ . We have a bundle decomposition  $\Lambda^2 TN = \Lambda_+^2 TN \oplus \Lambda_-^2 TN$ , and the restriction of  $\hat{h}$  on each of  $\Lambda_\pm^2 TN$  has signature  $(1, 2)$ . The space-like (or hyperbolic) twistor spaces associated with  $N$  are fiber bundles in  $\Lambda_\pm^2 TN$  such that fibers are hyperboloids of two sheets, and denoted by  $U_+(\Lambda_\pm^2 TN)$ . We can refer to [3] for space-like twistor spaces. Let  $M$  be a Riemann surface and  $F : M \rightarrow N$  a space-like and conformal immersion with zero mean curvature vector. Then  $F$  gives its space-like twistor lifts, which are sections of  $U_+(\Lambda_\pm^2 F^*TN)$ . Let  $Q$  be a complex quartic differential on  $M$  defined by  $F$  as in the previous paragraph. Then isotropy of  $F$  is given by  $Q \equiv 0$ , which is equivalent to horizontality of a space-like twistor lift of  $F$  ([1]).

Let  $(N, h)$  be as in the previous paragraph. The time-like twistor spaces associated with  $N$  are fiber bundles in  $\Lambda_\pm^2 TN$  such that fibers are hyperboloids of one sheet, and denoted by  $U_-(\Lambda_\pm^2 TN)$ . We can refer to [7], [8] for time-like twistor spaces. Let  $M$  be a Lorentz surface, which is an analogue of a Riemann surface and a two-dimensional manifold equipped with a holomorphic system of paracomplex coordinate neighborhoods. Let  $F : M \rightarrow N$  be a time-like and conformal immersion with zero mean curvature vector. Then  $F$  gives its time-like twistor lifts  $\Theta_{F,\pm}$ , which are sections of  $U_-(\Lambda_\pm^2 F^*TN)$ . Let  $Q$  be a paracomplex quartic differential on  $M$  defined by  $F$ . Then isotropy of  $F$  is given by  $Q \equiv 0$ . If one of  $\Theta_{F,\pm}$  is horizontal, then  $Q \equiv 0$  ([1]), while  $Q \equiv 0$  does not necessarily mean the horizontality of  $\Theta_{F,\pm}$ : it is possible that although  $F$  is isotropic, the covariant derivatives of  $\Theta_{F,\pm}$  are not zero but light-like. The covariant derivatives of  $\Theta_{F,\pm}$  are light-like or zero if and only if one of the following holds: (a) the shape operator of a light-like normal vector field vanishes and then  $Q$  vanishes; (b) the shape operator of any normal vector field is light-like or zero, and then  $Q$  is null or zero ([2]). The conformal Gauss maps of time-like surfaces of Willmore type in 3-dimensional Lorentzian space forms with zero holomorphic quartic differential satisfy Condition (a) ([1]). If  $N$  is a 4-dimensional neutral space form, then we can characterize surfaces with Condition (b), based on the Gauss-Codazzi-Ricci equations ([2]).

## REFERENCES

- [1] N. Ando, Surfaces with zero mean curvature vector in neutral 4-manifolds, *Diff. Geom. Appl.* **72** (2020) 101647.
- [2] N. Ando, The lifts of surfaces in neutral 4-manifolds into the 2-Grassmann bundles, preprint.
- [3] D. Blair, J. Davidov and O. Muškarov, Hyperbolic twistor spaces, *Rocky Mountain J. Math.* **35** (2005) 1437–1465.
- [4] R. Bryant, Conformal and minimal immersions of compact surfaces into the 4-sphere, *J. Differential Geom.* **17** (1982) 455–473.
- [5] J. Eells and S. Salamon, Twistorial construction of harmonic maps of surfaces into four-manifolds, *Annali della Scuola Normale Superiore di Pisa, Classe di Scienze* **12** (1985) 589–640.
- [6] T. Friedrich, On surfaces in four-spaces, *Ann. Glob. Anal. Geom.* **2** (1984) 257–287.
- [7] K. Hasegawa and K. Miura, Extremal Lorentzian surfaces with null  $\tau$ -planar geodesics in space forms, *Tohoku Math. J.* **67** (2015) 611–634.
- [8] G. Jensen and M. Rigoli, Neutral surfaces in neutral four-spaces, *Matematiche (Catania)* **45** (1990) 407–443.

## Table of contents

<b>L. M. Alabdulsada, L. Kozma</b> <i>Hopf-Rinow theorem of sub-Finslerian geometry</i>	<b>2</b>
<b>Y. Aliyev</b> <i>Geometric properties of interception curves</i>	<b>3</b>
<b>M. Amram</b> <i>Planar and non-planar degenerations with related fundamental groups</i>	<b>5</b>
<b>N. Ando</b> <i>Surfaces with zero mean curvature vector in 4-dimensional spaces</i>	<b>6</b>
<b>B. Apanasov</b> <i>Dynamics in nilpotent groups and deformations of locally symmetric rank one manifolds</i>	<b>8</b>
<b>M. Atteya</b> <i>Characterizing Linear Mappings Through Unital Algebra</i>	<b>9</b>
<b>S. Sharma, V. K. Bhat</b> <i>Edge resolvability and topological characteristics of zero-divisor graphs</i>	<b>10</b>
<b>V. Bilet, O. Dovgoshey</b> <i>From minimality to maximality via metric reflection</i>	<b>12</b>
<b>D. Bolotov</b> <i>Thurston norm and Euler classes of bounded mean curvature foliations on hyperbolic 3-Manifolds</i>	<b>14</b>
<b>A. Bolsinov</b> <i>Nijenhuis geometry and its applications</i>	<b>15</b>
<b>E. Bonacci</b> <i>Shape optimization in the batch crystallization of CAM</i>	<b>17</b>
<b>F. Bulnes</b> <i>Homotopies to Diffeomorphisms in Symplectic Field Theory</i>	<b>18</b>
<b>D. Cheban</b> <i>Global asymptotic stability of generalized homogeneous dynamical systems</i>	<b>19</b>
<b>Y. Cherevko, V. Berezovski, J. Mikeš, Y. Fedchenko</b> <i>Hyper-holomorphically projective mappings of hyper-Kähler manifolds</i>	<b>21</b>
<b>S. Dann</b> <i>On a problem of Fejes Toth</i>	<b>22</b>
<b>M. Golasinski, T. de Melo, R. Bononi</b> <i>Gottlieb groups of some Moore spaces</i>	<b>23</b>
<b>A. Białyzyt, A. Denkowska, M. P. Denkowski</b> <i>Inner semi-continuity of medial axes and conflict sets</i>	<b>24</b>
<b>K. v. Dichter</b> <i>The diameter-width-ratio for complete and pseudo-complete sets</i>	<b>25</b>
<b>O. Dovhopiatyi</b> <i>On the possibility of joining two pairs of points in convex domains using paths</i>	<b>27</b>
<b>Yu. Drozd</b> <i>Backström curves</i>	<b>28</b>