



International
Scientific Conference

Algebraic and Geometric Methods of Analysis

26-30 may 2020
Odesa, Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odessa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Odessa I. I. Mechnikov National University
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

PROGRAM COMMITTEE

Chairman: Prishlyak A. (<i>Kyiv, Ukraine</i>)	Kiosak V. (<i>Odesa, Ukraine</i>)	Pokas S. (<i>Odesa, Ukraine</i>)
Balan V. (<i>Bucharest, Romania</i>)	Kirillov V. (<i>Odesa, Ukraine</i>)	Polulyakh E. (<i>Kyiv, Ukraine</i>)
Banakh T. (<i>Lviv, Ukraine</i>)	Konovenko N. (<i>Odesa, Ukraine</i>)	Sabitov I. (<i>Moscow, Russia</i>)
Bolotov D. (<i>Kharkiv, Ukraine</i>)	Lyubashenko V. (<i>Kyiv, Ukraine</i>)	Savchenko A. (<i>Kherson, Ukraine</i>)
Borysenko O. (<i>Kharkiv, Ukraine</i>)	Maksymenko S. (<i>Kyiv, Ukraine</i>)	Sergeeva A. (<i>Odesa, Ukraine</i>)
Cherevko Ye. (<i>Odesa, Ukraine</i>)	Matsumoto K. (<i>Yamagata, Japan</i>)	Shelekhov A. (<i>Tver, Russia</i>)
Fedchenko Yu. (<i>Odesa, Ukraine</i>)	Mormul P. (<i>Warsaw, Poland</i>)	Volkov V. (<i>Odesa, Ukraine</i>)
Karlova O. (<i>Chernivtsi, Ukraine</i>)	Mykhailyuk V. (<i>Chernivtsi, Ukraine</i>)	Zarichnyi M. (<i>Lviv, Ukraine</i>)
	Plachta L. (<i>Krakov, Poland</i>)	

ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Svytyy I., Dean of the Faculty of Computer Systems and Automation.

ORGANIZING COMMITTEE

Kirillov V.
Konovenko N.
Fedchenko Yu.

Maksymenko S.
Cherevko Ye.

Osadchuk E.
Prus A.

ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБІТНИЦТВА

Centrally extended generalization of the superconformal loop Lie algebra and integrable heavenly type systems on supermanifolds

Oksana Ye. Hentosh

(Pidstryhach Inst. for Applied Problems of Mech. and Math., NASU, Lviv, Ukraine)

E-mail: ohen@ua.fm

Let us consider the semi-direct sum $\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}_{reg}^*$ of the loop Lie algebra $\tilde{\mathcal{G}} := \widetilde{diff}(\mathbb{T}^{1|N})$, consisting of the superconformal vector fields on a supertor $\mathbb{T}^{1|N}$ in the forms:

$$\tilde{a} := a\partial/\partial x + \frac{1}{2} \sum_{i=1}^N (D_{\vartheta_i} a) D_{\vartheta_i}, \quad a := a(x, \vartheta; \lambda), \quad (1)$$

where $a \in C^\infty(\mathbb{T}^{1|N} \times (\mathbb{D}_+^1 \cup \mathbb{D}_-^1); \Lambda_0)$, $(x, \vartheta) \in \mathbb{T}^{1|N} \simeq \mathbb{S}^1 \times \Lambda_1^N$, $\Lambda := \Lambda_0 \oplus \Lambda_1$ is a infinite-dimensional Grassmann algebra over $\mathbb{C} \supset \Lambda_0$, $\vartheta := (\vartheta_1, \vartheta_2, \dots, \vartheta_N)$ and $D_{\vartheta_i} := \partial/\partial\vartheta_i + \vartheta_i\partial/\partial x$, $i = \overline{1, N}$, which are holomorphic in the "spectral" parameter $\lambda \in \mathbb{C}$ on the interior $\mathbb{D}_+^1 \subset \mathbb{C}$ and exterior $\mathbb{D}_-^1 \subset \mathbb{C}$ regions of the unit centrally located disk $\mathbb{D}^1 \subset \mathbb{C}$, and its regular dual space $\tilde{\mathcal{G}}_{reg}^*$ with respect to the parity:

$$(\tilde{l}, \tilde{a})_0 = \text{res } \lambda^{-1} \int_{\mathbb{T}^{1|N}} dx d^N \vartheta (la), \quad \tilde{l} := l(x, \vartheta; \lambda)(dx + \sum_{i=1}^N \vartheta_i d\vartheta_i) \in \tilde{\mathcal{G}}_{reg}^*, \quad (2)$$

where $l \in C^\infty(\mathbb{T}^{1|(2k-1)} \times (\mathbb{D}_+^1 \cup \mathbb{D}_-^1); \Lambda_1)$ if $N = 2k - 1$ and $l \in C^\infty(\mathbb{T}^{1|2k} \times (\mathbb{D}_+^1 \cup \mathbb{D}_-^1); \Lambda_0)$ if $N = 2k$, $k \in \mathbb{N}$. The superconformal loop Lie algebra $\tilde{\mathcal{G}}$ possesses the commutator:

$$[\tilde{a}, \tilde{b}] = \tilde{c}, \quad \tilde{c} := c\partial/\partial x + \frac{1}{2} \sum_{i=1}^N (D_{\vartheta_i} c) D_{\vartheta_i},$$

$$c := a(\partial b/\partial x) - b(\partial a/\partial x) + \frac{1}{2} \sum_{i=1}^N (D_{\vartheta_i} a)(D_{\vartheta_i} b), \quad \tilde{a}, \tilde{b} \in \tilde{\mathcal{G}},$$

splits into the direct sum of its Lie subalgebras $\tilde{\mathcal{G}} = \tilde{\mathcal{G}}_+ \oplus \tilde{\mathcal{G}}_-$, for which the following dual spaces are identified: $\tilde{\mathcal{G}}_{+,reg}^* \simeq \tilde{\mathcal{G}}_-$, $\tilde{\mathcal{G}}_{-,reg}^* \simeq \tilde{\mathcal{G}}_+$. Here $\tilde{a}(\infty) = 0$ for any $\tilde{a}(\lambda) \in \tilde{\mathcal{G}}_-$. On $\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}_{reg}^*$ one determines the commutator:

$$[\tilde{a} \times \tilde{l}, \tilde{b} \times \tilde{m}] := [\tilde{a}, \tilde{b}] \times (ad_a^* \tilde{m} - ad_b^* \tilde{l}), \quad \tilde{a}, \tilde{b} \in \tilde{\mathcal{G}}, \quad \tilde{l}, \tilde{m} \in \tilde{\mathcal{G}}_{reg}^*,$$

where ad^* is the co-adjoint action of $\tilde{\mathcal{G}}$ with respect to the parity (2) and

$$ad_a^* l = l_x a + \frac{4-N}{2} l a_x + \frac{(-1)^{N+1}}{2} \sum_{i=1}^N (D_{\vartheta_i} l)(D_{\vartheta_i} a)$$

for any vector field $\tilde{a} \in \tilde{\mathcal{G}}$ and a fixed element $\tilde{l} \in \tilde{\mathcal{G}}_{reg}^*$, as well as nondegenerate symmetric bilinear form:

$$(\tilde{a} \times \tilde{l}, \tilde{b} \times m) = (\tilde{l}, \tilde{b})_0 + (\tilde{m}, \tilde{a})_0.$$

One constructs the central extension $\tilde{\mathfrak{G}} := \tilde{\mathfrak{G}} \oplus \mathbb{C}$ of the Lie algebra $\tilde{\mathfrak{G}} := \prod_{z \in \mathbb{S}^1} (\tilde{\mathcal{G}} \ltimes \tilde{\mathcal{G}}_{reg}^*)$ by the 2-cocycle [1]:

$$\omega_2(\tilde{a} \times \tilde{l}, \tilde{b} \times m) = \int_{\mathbb{S}^1} dz ((\tilde{l}, \partial \tilde{b}/\partial z)_0 - (\tilde{m}, \partial \tilde{a}/\partial z)_0), \quad (\tilde{a} \times \tilde{l}), (\tilde{b} \times \tilde{m}) \in \tilde{\mathfrak{G}}, \quad z \in \mathbb{S}^1.$$

The Lie algebra $\tilde{\mathfrak{G}}$ permits the standard splitting $\tilde{\mathfrak{G}} := \tilde{\mathfrak{G}}_+ \oplus \tilde{\mathfrak{G}}_-$ of the Lie algebra $\tilde{\mathfrak{G}}$ into the direct sum of its Lie subalgebras $\tilde{\mathfrak{G}}_+ := \prod_{z \in \mathbb{S}^1} (\tilde{\mathcal{G}}_+ \ltimes \tilde{\mathcal{G}}_{-,reg}^*)$ and $\tilde{\mathfrak{G}}_- := \prod_{z \in \mathbb{S}^1} (\tilde{\mathcal{G}}_- \ltimes \tilde{\mathcal{G}}_{+,reg}^*)$. Thus, by means of the \mathcal{R} -operator approach [2] one introduces the following Lie-Poisson bracket:

$$\{\mu, \nu\}_{\mathcal{R}} = (\tilde{a} \times \tilde{l}, [R\nabla_r \mu(\tilde{a} \times \tilde{l}), \nabla_l \nu(\tilde{a} \times \tilde{l})] + [\nabla_r \mu(\tilde{a} \times \tilde{l}), R\nabla_l \nu(\tilde{a} \times \tilde{l})]) +$$

$$+ \omega_2(R\nabla_r \mu(\tilde{a} \times \tilde{l}), \nabla_l \nu(\tilde{a} \times \tilde{l})) + \omega_2(\nabla_r \mu(\tilde{a} \times \tilde{l}), R\nabla_l \nu(\tilde{a} \times \tilde{l})), \quad (3)$$

where $\mu, \nu \in \mathcal{D}(\tilde{\mathfrak{G}}^*)$ are arbitrary smooth by Frechet functionals on $\tilde{\mathfrak{G}}^*$, $\mathcal{R} = (P_+ - P_-)/2$, P_+ and P_- are projectors on $\tilde{\mathfrak{G}}_+$ and $\tilde{\mathfrak{G}}_-$ respectively, on the dual space $\tilde{\mathfrak{G}}^* \simeq \tilde{\mathfrak{G}}$ to the Lie algebra $\tilde{\mathfrak{G}}$. Here $\nabla_l h(\tilde{a} \times \tilde{l}) := (\nabla_l h_{\tilde{l}} \times \nabla_l h_{\tilde{a}}) \in \tilde{\mathfrak{G}}$ and $\nabla_r h(\tilde{a} \times \tilde{l}) := (\nabla_r h_{\tilde{l}} \times \nabla_r h_{\tilde{a}}) \in \tilde{\mathfrak{G}}$ are left and right gradients of any smooth functional $h \in \mathcal{D}(\tilde{\mathfrak{G}}^*)$ at a point $(\tilde{a} \times \tilde{l}) \in \tilde{\mathfrak{G}}^*$. Due to the Adler-Kostant-Symes theory [2] the Lie-Poisson bracket (3) generates the hierarchy of Hamiltonian flows:

$$\partial(\tilde{a} \times \tilde{l})/\partial t_p := -ad_{P_+ \nabla_l h^{(p)}(\tilde{a} \times \tilde{l})}^*(\tilde{a} \times \tilde{l}) = \{\tilde{a} \times \tilde{l}, h^{(p)}\}_{\mathcal{R}}, \quad (\tilde{a} \times \tilde{l}) \in \tilde{\mathfrak{G}}^*, \quad p \in \mathbb{Z}_+,$$

where $P_+ \nabla_l h^{(p)}(\tilde{a} \times \tilde{l}) = (\nabla_l h_{\tilde{l},+}^{(p)} \times \nabla_l h_{\tilde{a},+}^{(p)})$, $\nabla_l h^{(p)}(\tilde{a} \times \tilde{l}) = \lambda^p \nabla_l h(\tilde{a} \times \tilde{l})$, $\nabla_l h_{\tilde{l}} \sim \sum_{j \in \mathbb{Z}_+} \nabla_l h_{\tilde{l},j} \lambda^{-j}$ and $\nabla_l h_{\tilde{a}} \sim \sum_{j \in \mathbb{Z}_+} \nabla_l h_{\tilde{a},j} \lambda^{-j}$ as $|\lambda| \rightarrow \infty$, for any Casimir invariant $h \in I(\tilde{\mathfrak{G}}^*)$, satisfying, by definition, the following relationship:

$$ad_{\nabla_l h(\tilde{a} \times \tilde{l})}^*(\tilde{a} \times \tilde{l}) = 0, \quad (\tilde{a} \times \tilde{l}) \in \tilde{\mathfrak{G}}^*.$$

Any two Hamiltonian flows on $\tilde{\mathfrak{G}}^*$ in the forms:

$$\partial(\tilde{a} \times \tilde{l})/\partial y = \{\tilde{a} \times \tilde{l}, h^{(y)}(\tilde{a} \times \tilde{l})\}_{\mathcal{R}}, \quad \partial(\tilde{a} \times \tilde{l})/\partial t = \{\tilde{a} \times \tilde{l}, h^{(t)}(\tilde{a} \times \tilde{l})\}_{\mathcal{R}},$$

where $\nabla_l h^{(y)} = \lambda^{p_y} \nabla_l h(\tilde{a} \times \tilde{l})$, $\nabla_l h^{(t)} = \lambda^{p_t} \nabla_l h(\tilde{a} \times \tilde{l})$, $p_y, p_t \in \mathbb{Z}_+$, and $h \in I(\tilde{\mathfrak{G}}^*)$, give rise to the separately commuting evolution equations:

$$\partial \tilde{a} / \partial y = -[\nabla_l h_{\tilde{l},+}^{(y)}, \tilde{a}] + \partial(\nabla_l h_{\tilde{l},+}^{(y)}) / \partial z, \quad \partial \tilde{a} / \partial t = -[\nabla_l h_{\tilde{l},+}^{(t)}, \tilde{a}] + \partial(\nabla_l h_{\tilde{l},+}^{(t)}) / \partial z, \quad (4)$$

and

$$\begin{aligned} \partial \tilde{l} / \partial y &= -ad_{\nabla_l h_{\tilde{l},+}^{(y)}}^* \tilde{l} + ad_{\tilde{a}}^* \nabla_l h_{\tilde{a},+}^{(y)} + \partial(\nabla_l h_{\tilde{a},+}^{(y)}) / \partial z, \\ \partial \tilde{l} / \partial t &= -ad_{\nabla_l h_{\tilde{l},+}^{(t)}}^* \tilde{l} + ad_{\tilde{a}}^* \nabla_l h_{\tilde{a},+}^{(t)} + \partial(\nabla_l h_{\tilde{a},+}^{(t)}) / \partial z. \end{aligned}$$

Proposition 1. *The commutativity of evolutions (4) is equivalent to the relationship:*

$$[\nabla_l h_{\tilde{l},+}^{(y)}, \nabla_l h_{\tilde{l},+}^{(t)}] - \partial(\nabla_l h_{\tilde{l},+}^{(y)}) / \partial t + \partial(\nabla_l h_{\tilde{l},+}^{(t)}) / \partial y = 0, \quad (5)$$

which is reduced on every coadjoint orbit of the Lie algebra $\hat{\mathfrak{G}}$ to the Lax-Sato representation for some system of nonlinear heavenly type equations on a functional supermanifold. The relationship (5) is a compatibility condition for the following linear vector equations:

$$\partial \psi / \partial y + \nabla_l h_{\tilde{l},+}^{(y)} \psi = 0, \quad \partial \psi / \partial z + \tilde{a} \psi = 0, \quad \partial \psi / \partial t + \nabla_l h_{\tilde{l},+}^{(t)} \psi = 0,$$

where $(y, t; \lambda, z, x, \theta) \in (\mathbb{R}^2 \times (\mathbb{C} \times \mathbb{S}^1 \times \mathbb{T}^{1N}))$ and $\psi \in C^2(\mathbb{R}^2 \times (\mathbb{C} \times \mathbb{S}^1 \times \mathbb{T}^{1N}); \mathbb{C})$.

By use of the Lax-Sato compatibility condition (5) one can construct integrable systems of heavenly type equations on functional supermanifolds, which can be considered as generalizations of Lax-Sato integrable superanalogs [3] of the Mikhalev-Pavlov heavenly type equation, choosing the smooth functions $a := \sum_{k=1}^{K-1} w_{k,x}(x, \theta) \lambda^k - \lambda^K$ and $l := \sum_{k=1}^{K-1} \xi_{k,x}(x, \theta) \lambda^k$, $K \in \mathbb{N}$, in (1) and (2) respectively.

REFERENCES

- [1] Valentin Ovsienko, Claude Roger. Looped cotangent Virasoro algebra and non-linear integrable systems in dimension $2 + 1$. *Communications in Mathematical Physics*, 273(2) : 357–378, 2007.
- [2] Ludwig D. Faddeev, Leon A. Takhtadjan. *Hamiltonian methods in the theory of solitons*, *Classics in mathematics*. Berlin, Heidelberg : Springer-Verlag, 2007.
- [3] Oksana Hentosh, Yarena Prykarpatsky Ya. The Lax-Sato integrable heavenly equations on functional supermanifolds and their Lie-algebraic structure. *European Journal of Mathematics*, 6(1) : 232–247, 2020.

ЗМІСТ

G. M. Abdishukurova, A. Ya. Narmanov <i>On the geometry of submersions</i>	3
B. N. Apanasov <i>Hyperbolic 4-cobordisms, Teichmuller spaces and quasiregular mappings in space</i>	5
Aymaz I., Kansu M. <i>Representation of gravi-electromagnetism using matrix algebra</i>	7
V. Bilet, O. Dovgoshey <i>Uniqueness of pretangent spaces at infinity</i>	9
Bolotov D. <i>Foliations of 3-manifolds with small module of mean curvature</i>	10
Bolsinov A. V. <i>On integrability of geodesic flows on 3-dimensional manifolds</i>	11
E. Bonacci <i>Algebraic and geometric questions about the EM helix</i>	12
Borisenko A. A., Sukhorebska D. D. <i>Geodesics on regular tetrahedra in spherical space</i>	13
F. Bulnes <i>Motivic hypercohomology solutions in field theory II</i>	14
I. Denega <i>Estimate of maximum of the products of inner radii of mutually non-overlapping domains</i>	16
A. Dudko, V. Pivovarchik <i>Inverse problem for tree of Stieltjes strings</i>	18
N. Glazunov <i>Formal groups and algebraic cobordism</i>	20
O. Gok <i>A note on tensor product of Archimedean vector lattices</i>	22
E. Gül. <i>Trace Regularization Problem On a Banach Space</i>	24
O. Ye. Hentosh <i>Centrally extended generalization of the superconformal loop Lie algebra and integrable heavenly type systems on supermanifolds</i>	26
B. Hladysh, A. Prishlyak <i>Structure of functions on an oriented 2-manifold with the boundary</i>	28
D. A. Juraev <i>The Cauchy problem for matrix factorizations of the Helmholtz equation in a multidimensional bounded domain</i>	30
A. Kachurovskii <i>Fejer Sums and the von Neumann Ergodic Theorem</i>	31
B. N. Khabibullin, R. R. Muryasov <i>Mixed volumes/areas and distribution of zeros of holomorphic functions</i>	33
B. Klishchuk, R. Salimov <i>On the behavior at infinity of one class of homeomorphisms</i>	35
A. Kravchenko, S. Maksymenko <i>Automorphisms of cellular divisions of 2-sphere induced by functions with isolated critical points</i>	37
A. Kushner, E. Kushner, R. Matviichuk <i>Dynamics and exact solutions of linear PDEs</i>	39