



International

Scientific

Conference

Algebraic and Geometric Methods of Analysis

May 24-27, 2022, Odesa, Ukraine

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

SCIENTIFIC COMMITTEE

Co-Chairs:

Maksymenko S.
(*Kyiv, Ukraine*)

Prishlyak A.
(*Kyiv, Ukraine*)

Balan V.
(*Bucharest, Romania*)

Fedchenko Yu.
(*Odesa, Ukraine*)

Matsumoto K.
(*Yamagata, Japan*)

Banakh T.
(*Lviv, Ukraine*)

Karlova O.
(*Chernivtsi, Ukraine*)

Mormul P.
(*Warsaw, Poland*)

Bolotov D.
(*Kharkiv, Ukraine*)

Kiosak V.
(*Odesa, Ukraine*)

Plachta L.
(*Krakov, Poland*)

Cherevko Ye.
(*Odesa, Ukraine*)

Konovenko N.
(*Odesa, Ukraine*)

Polulyakh Ye.
(*Kyiv, Ukraine*)

Savchenko O.
(*Kherson, Ukraine*)

ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONTU;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONTU;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONTU;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";

ORGANIZING COMMITTEE

Konovenko N.
Maksymenko S.

Fedchenko Yu.
Cherevko Ye.

Osadchuk Ye.
Sergeeva O.

Soroka Yu.

On ternary assymmetric medial top-quasigroups

Fedir Sokhatsky

(Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine)

E-mail: fmsokha@ukr.net

Iryna Fryz

(Vasyl' Stus Donetsk National University, Vinnytsia, Ukraine)

E-mail: iryna.fryz@ukr.net

Let Q be an m element set. A ternary operation f defined on Q is called *invertible* and the pair $(Q; f)$ is a *quasigroup* of the order m , if for every a, b of Q the terms $f(x, a, b)$, $f(a, x, b)$, $f(a, b, x)$ define permutations of Q . To each ternary quasigroup $(Q; f)$ of the order m there corresponds a Latin cube of order m , i.e., a 3-dimensional array on m distinct symbols from Q , each of which occurs exactly once in any line of the array.

A triplet (f_1, f_2, f_3) of ternary operations is called *orthogonal* [1], if for all $a_1, a_2, a_3 \in Q$ the system

$$\begin{cases} f_1(x_1, x_2, x_3) = a_1, \\ f_2(x_1, x_2, x_3) = a_2, \\ f_3(x_1, x_2, x_3) = a_3 \end{cases}$$

has a unique solution, i.e., superimposition of the corresponding cubes gives a cube such that every triplet of elements of Q appears exactly once in it.

Geometric interpretation of orthogonality is its relationships with geometric nets. This application is well-studied for binary operations and the respective k -nets, projective and affine planes (see for example [2], [3]). Relationships between t -tuples of orthogonal n -ary quasigroups of order m and (t, m, n) -nets were studied in [4], [5], [6]. The respective nets have the same combinatorial and algebraic properties.

For every permutation $\sigma \in S_4$ a σ -*parastrophe* ${}^\sigma f$ of an invertible ternary operation f is defined by

$${}^\sigma f(x_{1\sigma}, x_{2\sigma}, x_{3\sigma}) = x_{4\sigma} : \iff f(x_1, x_2, x_3) = x_4.$$

In particular, a σ -parastrophe is called:

- an *i -th division* if $\sigma = (i4)$ for $i = 1, 2, 3$;
- *principal* if $4\sigma = 4$.

Therefore, each ternary operation has at most $4! = 24$ parastrophes; among them $3! = 6$ principal parastrophes. An invertible operation and the respective quasigroup are called *assymmetric* if all its parastrophes are different. A quasigroup is called *totally parastrophic orthogonal (top-quasigroup)*, if each triplet of its different parastrophes are orthogonal. Binary assymmetric top-quasigroups were studied in [7], for ternary case the following statements are true.

Theorem 1 ([8]). *A quasigroup $(Q; f)$ is medial if and only if there exists an abelian group $(Q; +)$ such that*

$$f(x_1, x_2, x_3) = \varphi_1 x_1 + \varphi_2 x_2 + \varphi_3 x_3 + a, \tag{1}$$

where $\varphi_1, \varphi_2, \varphi_3$ are pairwise commuting automorphisms of $(Q; +)$ and $a \in Q$.

Theorem 2. Let $(Q; f)$ be a medial ternary quasigroup $(Q; f)$ with (1) and $\tau_1, \tau_2, \tau_3 \in S_4$. The parastrophes ${}^{\tau_1}f, {}^{\tau_2}f, {}^{\tau_3}f$ are orthogonal if and only if the determinant

$$\begin{vmatrix} \varphi_{1\tau_1} & \varphi_{2\tau_1} & \varphi_{3\tau_1} \\ \varphi_{1\tau_2} & \varphi_{2\tau_2} & \varphi_{3\tau_2} \\ \varphi_{1\tau_3} & \varphi_{2\tau_3} & \varphi_{3\tau_3} \end{vmatrix}$$

is an automorphism of the group $(Q; +)$, where $\varphi_4 := J$ and $J(x) := -x$.

Note, that the pairwise commuting automorphisms $\varphi_1, \varphi_2, \varphi_3, J$ generate a commutative subring K of the ring $\text{End}(Q; +)$. Let $\vec{\nu} := (\nu_1, \nu_2, \nu_3)$ be a triplet of injections of the set $\{1, 2, 3\}$ into the set $\{1, 2, 3, 4\}$. The polynomial

$$d_{\vec{\nu}}(\gamma_1, \gamma_2, \gamma_3, \gamma_4) := \begin{vmatrix} \gamma_{1\nu_1} & \gamma_{2\nu_1} & \gamma_{3\nu_1} \\ \gamma_{1\nu_2} & \gamma_{2\nu_2} & \gamma_{3\nu_2} \\ \gamma_{1\nu_3} & \gamma_{2\nu_3} & \gamma_{3\nu_3} \end{vmatrix}$$

over the commutative ring K will be called *invertible-valued* over a set $H \subseteq K$, if all its values are automorphisms of the group $(Q; +)$ when the variables $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ take their values in H .

Theorem 3. A ternary medial quasigroup $(Q; f)$ with (1) is a top-quasigroup if and only if each polynomial $d_{\vec{\nu}}$ is invertible-valued over the set $\{\varphi_1, \varphi_2, \varphi_3, \varphi_4\}$, where $\varphi_4 := J$.

Theorem 4 ([9]). A ternary medial asymmetric top-quasigroup over a cyclic group of the order m exists if and only if the least prime factor of m is greater than 19.

REFERENCES

- [1] Galina B. Belyavskaya, Gary L. Mullen. Orthogonal hypercubes and n -ary operations, *Quasigroups and Related System*, 13: 73–76, 2005.
- [2] V.D. Belousov. *Configurations in algebraic webs*. Kishinev: Stiintsa, 1979. (in Russian).
- [3] A.D. Keedwell, J. Dénes. *Latin Squares and their Applications*. Amsterdam: North Holland, 2015.
- [4] C.F. Laywine, G.L. Mullen and G. Whittle. D -Dimensional hypercubes and the Euler and MacNeish conjectures. *Monatsh. Math.*, 111: 223–238, 1995.
- [5] Maks A. Akiwis, Vladislav V. Goldberg. Algebraic aspects of web geometry. *Commentationes Mathematicae Universitatis Carolinae*, 41(2): 205–236, 2000.
- [6] F.M. Sokhatsky. About multiary webs. *International Conference “Morse theory and its applications” dedicated to the memory and 70th anniversary of Volodymyr Vasylyovych Sharko*, September 25–28, 2019: 47–48, 2019.
- [7] G.B. Belyavskaya, T.V. Popovich. Totally conjugate orthogonal quasigroups and complete graphs. *Fundamentalnaya i prikladnaya matematika*, 13 (8): 17–26, 2010. (Russian)
- [8] V.D. Belousov. *n -ary quasigroups*. Chishinau: Stiintsa, 1972. (in Russian)
- [9] F. Sokhatsky, Ie. Pirus. About parastrophically orthogonal quasigroups. *Book of extended abstracts of the International Mathematical Conference on Quasigroups and Loops “Loops’15”*, 28 June - 04 July 2015: 46–47, 2015.

T. Obikhod <i>The role of topological invariants in the study of the early evolution of the Universe</i>	33
I. Ovtsynov <i>O-spheroids in metric and linear normed spaces</i>	34
T. Podousova, N. Vashpanova <i>Infinitesimal deformations of surfaces of negative Gaussian curvature with a stationary Ricci tensor</i>	37
A. Prishlyak <i>Structures of optimal flows on the Boy's and Girl's surfaces</i>	38
V.M. Prokip <i>About solvability of the matrix equation $AX = B$ over Bezout domains</i>	39
N. Saouli, F. Zouyed <i>Regularization Method for a class of inverse problem</i>	42
H. Sinyukova <i>Broadening of some vanishing theorems of global character about holomorphically projective mappings of Kahlerian spaces to the noncompact but complete ones.</i>	44
A. Skryabina, P. Stegantseva <i>The weight of T_0-topologies on n-element set that consistent with close to the discrete topology on $(n - 1)$-element set</i>	45
F. Sokhatsky, I. Fryz <i>On ternary assymetric medial top-quasigroups</i>	46
Andrei Teleman <i>Extension theorems for holomorphic bundles on complex manifolds with boundary</i>	48
J. Ueki <i>Recent progress in Iwasawa theory of knots and links</i>	50
М. Гречнева, П. Стеганцева <i>Про тип грассманового образу поверхонь з плоскою нормальною зв'язністю простору Мінковського</i>	52
В. Кіосак, Л. Кусік, В. Ісаєв <i>Про існування гедезично симетричних псевдоріманових просторів</i>	53
І. М. Курбатова, М. І. Піструїл <i>Геометричні об'єкти, інваріантні відносно квазі-геодезичних відображень псевдо-ріманових просторів з узагальнено-рекурентною афінорною структурою</i>	54
В. О. Мозель <i>Автоморфні функції та алгебри двовимірних сингулярних інтегральних операторів</i>	55
М. І. Піструїл, І. М. Курбатова <i>Канонічні квазі-геодезичні відображення псевдо-ріманових просторів з рекурентно-параболічною структурою</i>	56
С. І. Покась, А. О. Ніколайчук <i>Геометрія наближення для простору афінної зв'язності</i>	58
А.Соловійов, І.Курбатова, Ю.Хабарова <i>Про ZF-планарні відображення псевдо-ріманових просторів</i>	59
Т. О. Єрьоміна, О. А. Поварова <i>Дослідження властивостей неперервних обмежених розв'язків систем нелінійних різницево-функціональних рівнянь у гіперболічному випадку</i>	60