International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Let $X \subseteq \mathbb{R}^m$, $Y \subseteq \mathbb{R}^n$ be algebraic sets. Write $[X, Y]$ for the set of homotopy classes of continuous maps and $[X, Y]_{alg}$ the subset of $[X, Y]$ represented by regular maps. One of the main purposes of the talk is to estimate the size of $\pi_m(\mathbb{S}^n)_{alg} = [\mathbb{S}^m, \mathbb{S}^n]_{alg}$ in $\pi_m(\mathbb{S}^n) = [\mathbb{S}^m, \mathbb{S}^n]$.

Basing on [1], [3] and [4], we aim to show:

Theorem 3. *If $k = 0, 1, \dots, 7$ then elements of $\pi_{n+k}(\mathbb{S}^n)$ can be represented by regular maps for $n \geq 1$.*

Next, we make use of [2] to show a homeomorphism $TG_{n,r}(K) \xrightarrow{\approx} \mathrm{Idem}_{n,r}(K)$ for the tangent bundle $TG_{n,r}(K)$ of $G_{n,r}(K)$ and $\mathrm{Idem}_{n,r}(K)$, the set of all idempotent $n \times n$ matrices with rank r for $K = \mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$. Finally, we present:

Theorem 4. *If $K = \mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$ then there is:*

(1) *a regular deformation retraction* $\mathrm{Idem}_{n,r}(K) \to G_{n,r}(K)$*;*

(2) *an injection $\mathcal{P}_{\mathbb{C}}[V_{\mathbb{C}}, \mathrm{Idem}_{n,r}(K)] \to \mathcal{R}_{\mathbb{R}}[V, G_{n,r}(K)]$ from the sets of homotopy classes of complex-valued polynomial to such a set of real-valued regular maps, where $V_{\mathbb{C}}$ denotes the Zariski closure in the affine space $\mathbb{C}^n$ of a subset $V \subseteq \mathbb{R}^n$.*

References

[1] J. Bochnak, W. Kucharz, *Realization of homotopy classes by algebraic mappings*, J. Reine Angew. Math. **377**, 159-169 (1987).

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[3] R. Wood, *Polynomial Maps from Spheres to Spheres*, Invent. Math. **5**, 163-168 (1968).

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On homomorphisms of bicyclic extensions of archimedean totally ordered groups

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We follow the terminology of [1, 2]. Let G^+ be the positive cone of a totally ordered group. On the set $\mathscr{B}^+(G)) = G^+ \times G^+$ we define the semigroup operation "$\cdot$" in the following way

$$(a, b) \cdot (c, d) = \begin{cases} (c \cdot b^{-1} \cdot a, d), & \text{if } b < c; \\ (a, d), & \text{if } b = c; \\ (a, b \cdot c^{-1} \cdot d), & \text{if } b > c, \end{cases}$$

for $a, b, c, d \in G^+$.

Theorem 1. *Let G and H be archimedean totally ordered groups. Then every o-homomorphism $\widehat{\varphi} \colon G \to H$ generates a monoid homomorphism $\widetilde{\varphi} \colon \mathscr{B}^+(G) \to \mathscr{B}^+(H)$, and every monoid homomorphism $\widetilde{\varphi} \colon \mathscr{B}^+(G) \to \mathscr{B}^+(H)$ generates an o-homomorphism $\widehat{\varphi} \colon G \to H$, which agree* *according to the formula*

$$(x,y)\widetilde{\varphi} = \big((x)\widehat{\varphi}, (y)\widehat{\varphi}\big), \qquad x, y \in G^+.$$

Theorem 2. *Let G be an archimedean totally ordered group. Then the semigroup $\mathbf{End}^o(G)$ of o-endomorphisms of G is isomorphic to the semigroup $\mathbf{End}(\mathscr{B}^+(G))$ of endomorphisms of the monoid $\mathscr{B}^+(G)$.*

We define the category $\mathfrak{TOAG}$ by

(1) $\mathbf{Ob}(\mathfrak{TOAG}) = \{G\colon G$ is an archimedean totally ordered group$\}$;
(2) $\mathbf{Mor}(\mathfrak{TOAG})$ are o-homomorphisms of archimedean totally ordered groups,

and the category $\mathfrak{BETOAG}$ in the following way

(1) $\mathbf{Ob}(\mathfrak{BETOAG})$ are bicyclic extensions $\mathscr{B}^+(G)$ of archimedean totally ordered groups $G \in \mathbf{Ob}(\mathfrak{TOAG})$;
(2) $\mathbf{Mor}(\mathfrak{BETOAG})$ are homomorphisms of monoids $\mathscr{B}^+(G) \in \mathbf{Ob}(\mathfrak{BETOAG})$.

Theorem 3. *The categories $\mathfrak{TOAG}$ and $\mathfrak{BETOAG}$ are isomorphic.*

References

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The Interaction of an Infinite Number of Eddy Flows

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The Boltzmann kinetic equation plays an important role in the kinetic theory of gases. In paper [2], we consider this equation for a model of hard spheres that describes particles of any gas which move translationally with a certain linear velocity, collide by the laws of classical mechanics and can not rotate. For this model, the equation has the form [1]

$$D(f) = Q(f, f), \tag{1}$$

$$D(f) \equiv \frac{\partial f}{\partial t} + \left(V, \frac{\partial f}{\partial x}\right), \tag{2}$$

$$Q(f, f) \equiv \frac{d^2}{2} \int_{R^3} dV_1 \int_{\Sigma} d\alpha |(V - V_1, \alpha)| \times \Big[f(t, x, V_1')f(t, x, V') - f(t, x, V)f(t, x, V_1)\Big], \tag{3}$$

and V, V_1, V', V_1' are the velocities of particles before and after collision, respectively, determined by the relations

$$V' = V - \alpha(V - V_1, \alpha),$$
$$V_1' = V_1 + \alpha(V - V_1, \alpha).$$