International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Bolotov D.** (*Kharkiv, Ukraine*)
- **Bondarenko V.** (*Kyiv, Ukraine*)
- **Boychuk O.** (*Kyiv, Ukraine*)
- **Boyko V.** (*Kyiv, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Dorogovtsev A.** (*Kyiv, Ukraine*)
- **Drozd Yu.** (*Kyiv, Ukraine*)
- **Gerasymenko V.** (*Kyiv, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Kiosak V.** (*Odesa, Ukraine*)
- **Kochubei A.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Mikhailets V.** (*Kyiv, Ukraine*)
- **Ostrovskyi V.** (*Kyiv, Ukraine*)
- **Petravchuk A.** (*Kyiv, Ukraine*)
- **Plaksa S.** (*Kyiv, Ukraine*)
- **Portenko M.** (*Kyiv, Ukraine*)
- **Pratsiovytyi M.** (*Kyiv, Ukraine*)
- **Savchenko O.** (*Kherson, Ukraine*)
- **Romanyuk A.** (*Kyiv, Ukraine*)
- **Timokha O.** (*Kyiv, Ukraine*)

Organizing commitee

- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Osadchuk Ye.** (*Odesa, Ukraine*)
- **Sergeeva O.** (*Odesa, Ukraine*)

then the following inequalities hold

$$\frac{1}{\sqrt{\pi}}e^{-\alpha n^r}\left(1-\frac{2\gamma_{\alpha,r,n}e^{-2\alpha r(n-1)^{r-1}}}{1+2\gamma_{\alpha,r,n}e^{-2\alpha r(n-1)^{r-1}}}\right)^{\frac{1}{2}} \leq P_{2n}(C^{\alpha,r}_{\bar\beta,2},C)$$

$$\leq P_{2n-1}(C^{\alpha,r}_{\bar\beta,2},C) \leq \frac{1}{\sqrt{\pi}}e^{-\alpha n^r}\left(1+e^{-2\alpha r n^{r-1}}\left(1+\frac{1}{2\alpha r n^{r-1}}\right)\right)^{\frac{1}{2}}, \tag{3}$$

where P_N is any of the widths b_N, d_N, λ_N or π_N and

$$\gamma_{\alpha,r,n} = \left(1+\frac{1}{\alpha r(n-1)^{r-1}}+e^{-2\alpha(n-1)^r}\max\left\{e^{4\alpha},\frac{e^2}{\alpha^{1+1/r}}\right\}\right). \tag{4}$$

Theorem 2. *Let $\bar\beta=\{\beta_k\}_{k=1}^{\infty}, \beta_k\in\mathbb{R}$, $\alpha>0$, $r>1$, $n\in\mathbb{N}$ and the condition (2) is satisfied. Then as $n\to\infty$ the following asymptotic equalities hold*

$$\left.\begin{array}{l} P_{2n}(C^{\alpha,r}_{\bar\beta,2},C) \\ P_{2n-1}(C^{\alpha,r}_{\bar\beta,2},C)\end{array}\right\} = e^{-\alpha n^r}\left(\frac{1}{\sqrt{\pi}}+\mathcal{O}(1)\gamma_{\alpha,r,n}e^{-\alpha r(n-1)^{r-1}}\right), \tag{5}$$

where P_N is any of the widths b_N, d_N, λ_N or π_N and $\gamma_{\alpha,r,n}$ is defined by (4) and $\mathcal{O}(1)$ are the quantities uniformly bounded in all parameters.

Note that the Theorem 2 complements the results of the works of Shevaldin (1992), Stepanets and Serdyuk (1995), Serdyuk (1999), Serdyuk and Sokolenko (2011), Serdyuk and Bodenchuk (2013), which contain exact estimates for the widths of the classes of convolutions with classical or generalized Poisson kernels.

This work was partially supported by the Grant H2020-MSCA-RISE-2019, project number 873071 (SOMPATY: Spectral Optimization: From Mathematics to Physics and Advanced Technology).

References

[1] A.I. Stepanets. *Methods of Approximation Theory.* Utrecht: VSP, 2005.

Poincaré-Reeb graphs of real algebraic domains

Arnaud Bodin
(Université de Lille, CNRS, Laboratoire Paul Painlevé, Lille, France)
E-mail: `arnaud.bodin@univ-lille.fr`

Patrick Popescu-Pampu
(Université de Lille, CNRS, Laboratoire Paul Painlevé, Lille, France)
E-mail: `patrick.popescu-pampu@univ-lille.fr`

Miruna-Stefana Sorea
(SISSA - Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italy and Lucian Blaga University of Sibiu, Romania)
E-mail: `mirunastefana.sorea@sissa.it`

An *algebraic domain* is a closed topological subsurface of a real affine plane whose boundary consists of disjoint smooth connected components of real algebraic plane curves. We study the geometric shape of an algebraic domain by collapsing all vertical segments contained in it:

this yields a *Poincaré–Reeb graph*, which is naturally transversal to the foliation by vertical lines. We show that any transversal graph whose vertices have only valencies 1 and 3 and are situated on distinct vertical lines can be realized as a Poincaré–Reeb graph.

References

[1] Miruna-Stefana Sorea. The shapes of level curves of real polynomials near strict local minima. *Ph.D. thesis, Université de Lille/Laboratoire Paul Painlevé*, France, 2018

[2] Miruna-Stefana Sorea. Constructing separable Arnold snakes of Morse polynomials. *Portugaliae Mathematica. A Journal of the Portuguese Mathematical Society* 77 (2020), no. 2, 219–260.

[3] Miruna-Stefana Sorea. Measuring the local non-convexity of real algebraic curves. *Journal of Symbolic Computation* 109 (2022), 482–509.

[4] Miruna-Stefana Sorea. Permutations encoding the local shape of level curves of real polynomials via generic projections. *Annales de l'Institut Fourier (Grenoble)* 72 (2022), no. 4, 1661–1703.

[5] Arnaud Bodin, Patrick Popescu-Pampu, Miruna-Stefana Sorea. Poincaré-Reeb graphs of real algebraic domains. https://arxiv.org/abs/2207.06871 (to appear in *Revista Matemática Complutense*)

On univalent trinomials

Dmytro Dmytryshyn
(Odessa National Polytechnic University)
E-mail: dmitrishin@op.edu.ua

Daniel Gray
(Georgia Southern University)
E-mail: dagray@georgiasouthern.edu

Oleksandr Stokolos
(Georgia Southern University)
E-mail: astokolos@georgiasouthern.edu

The Suffridge polynomials were introduced by T. Suffridge [1] and play an important role in complex analysis. Suffridge polynomials are closely related to the Brandt polynomials, first mentioned in M. Brandt's Ph.D. thesis [2] and rediscovered in [3].

The T-folded version of these polynomials were suggested in [4, 5] and several important conjectures about them were made.

In this talk we will outline the proof of these conjectures in the particular case of trinomials

$$z + az^{1+T} + bz^{1+2T}.$$

A beautiful geometry behind the scenes will be illuminated.

References

[1] T.J. Suffridge, On univalent polynomials, *J. London Math. Soc.* 44, 496–504, 1969.

[2] M. Brandt, Variationsmethoden für in der Einheitskreisscheibe schlichte Polynome, Th⊠esis, Humboldt-Univ. Berlin, 1987.

[3] D. Dmitrishin, A. Smorodin, and A. Stokolos, An extremal problem for polynomials, *Applied and Computational Harmonic Analysis*, 56: 283–305, 2022.

[4] D. Dmitrishin, D. Gray, and A. Stokolos, Some extremal problems for trinomials with fold symmetry, 12(4), *Analysis and Mathematical Physics*, 2022.

[5] D. Dmitrishin, D. Gray, and A. Stokolos, On the Koebe quarter theorem for trionomials with fold symmetry, *Proceedings of the AMS* (to appear).