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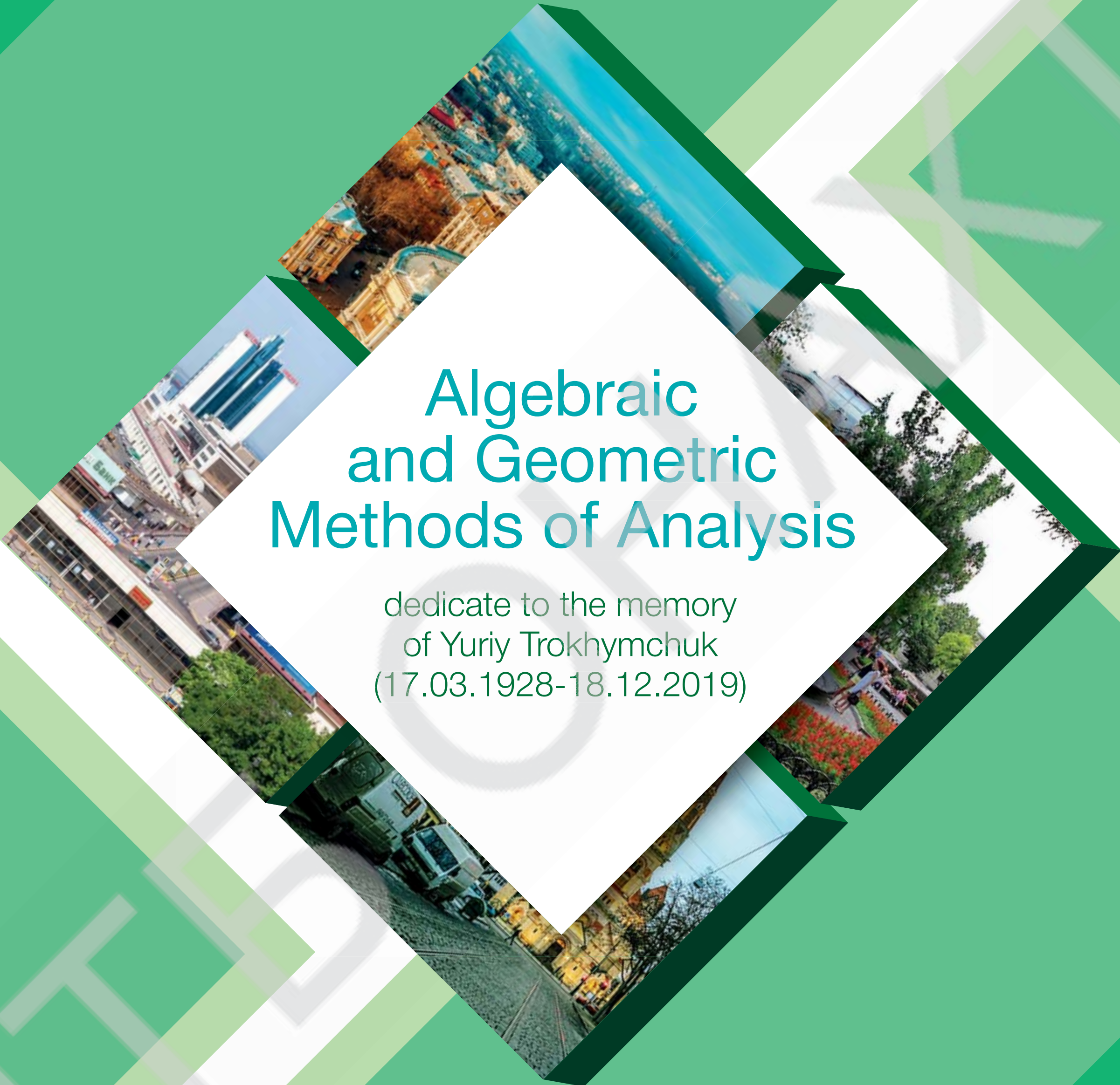

# Algebraic and Geometric Methods of Analysis

dedicate to the memory
of Yuriy Trokhymchuk
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## List of topics

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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# On the monoid of cofinite partial isometries of positive integers with a bounded finite noise

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We follow the terminology of [2, 4, 5]. For any positive integer $j$ the semigroup $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ is called the *monoid of cofinite isometries of positive integers with the noise* $j$. It was introduced in [4].

Any inverse semigroup $S$ admits the *minimum group congruence* $\mathfrak{C}_{\mathbf{mg}}$:

$$a\mathfrak{C}_{\mathbf{mg}}b \quad \text{if and only if} \quad \text{there exists} \quad e \in E(S) \quad \text{such that} \quad ea = eb.$$

**Proposition 1.** *Let $\gamma$ and $\delta$ be elements of the monoid $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$. Then $\gamma\mathfrak{C}_{\mathbf{mg}}\delta$ in $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ if and only if $n_{\gamma}^{\mathbf{r}} - n_{\gamma}^{\mathbf{d}} = n_{\delta}^{\mathbf{r}} - n_{\delta}^{\mathbf{d}}$. Moreover, the quotient semigroup $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}/\mathfrak{C}_{\mathbf{mg}}$ is isomorphic to the additive group of integers $\mathbb{Z}(+)$ by the map*

$$\pi_{\mathfrak{C}_{\mathbf{mg}}} \colon \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]} \to \mathbb{Z}(+), \quad \gamma \mapsto n_{\delta}^{\mathbf{r}} - n_{\delta}^{\mathbf{d}}.$$

**Example 2.** We put $\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]} = \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]} \sqcup \mathbb{Z}(+)$ and extend the multiplications from $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ and $\mathbb{Z}(+)$ onto $\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ in the following way:

$$k \cdot \gamma = \gamma \cdot k = k + (\gamma)\pi_{\mathfrak{C}_{\mathbf{mg}}} \in \mathbb{Z}(+), \qquad \text{for all} \quad k \in \mathbb{Z}(+) \quad \text{and} \quad \gamma \in \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}.$$

**Theorem 3.** *For any positive integer $j$ every Hausdorff shift-continuous topology $\tau$ on $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ is discrete.*

**Proposition 4.** *Let $j$ be any positive integer and $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ be a proper dense subsemigroup of a Hausdorff semitopological semigroup $S$. Then $I = S \setminus \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ is a closed ideal of $S$.*

**Theorem 5.** *Let $j$ be any positive integer and $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ be a proper dense subsemigroup of a Hausdorff topological inverse semigroup $S$. Then $I = S \setminus \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ is a topological group.*

**Corollary 6.** *Let $j$ be any positive integer and $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ be a proper dense subsemigroup of a Hausdorff topological inverse semigroup $S$. Then the group $S \setminus \mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ contains a dense cyclic subgroup.*

**Example 7.** Let $\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ be a semigroup defined in Example 2. Put $M$ be an arbitrary subset of $\{2, \ldots, j\}$. We define the topology $\tau_{\mathsf{lc}}^{M}$ on $\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ in the following way:

- $(i)$ all elements of the monoid $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ are isolated points in $\big(\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}, \tau_{\mathsf{lc}}^{M}\big)$;
- $(ii)$ for any $k \in \mathbb{Z}(+)$ the family $\mathscr{B}_{\mathsf{lc}}^{M}(k) = \big\{U_i^M(k) \colon i \in \mathbb{N}\big\}$, where

$$U_i^M(k) = \{k\} \cup \big\{\gamma \in \mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}[M] \colon k \preccurlyeq \gamma \text{ and } n_{\gamma}^{\mathbf{d}} \geqslant i\big\},$$

  is the base of the topology $\tau_{\mathsf{lc}}^{M}$ at the point $k \in \mathbb{Z}(+)$.

**Theorem 8.** *Let $j$ be any positive integer and $\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$ be a proper dense subsemigroup of a Hausdorff locally compact topological inverse semigroup $(S, \tau)$. Then $(S, \tau)$ topologically isomorphic to the topological inverse semigroup $\big(\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}, \tau_{\mathsf{lc}}^{M}\big)$ for some subset $M$ of $\{2, \ldots, j\}$.*

**Corollary 9.** *For any positive integer $j$ there exists $(j-1)!+1$ distinct topologically non-isomorphic Hausdorff locally compact semigroup inverse topologies on the monoid $\mathcal{C}\mathbf{I}\mathbb{N}_{\infty}^{\boldsymbol{g}[j]}$.*

The obtained results generalize the corresponding results of the papers [1] and [3].

## References


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