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**Algebraic  
and Geometric  
Methods of Analysis**

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## LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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# On the monoid of cofinite partial isometries of positive integers with a bounded finite noise

Oleg Gutik

(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)

*E-mail:* oleg.gutik@lnu.edu.ua

Pavlo Khylynskyi

(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)

*E-mail:* pavlo.khylynskyi@lnu.edu.ua

We follow the terminology of [2, 4, 5]. For any positive integer  $j$  the semigroup  $\mathbf{IN}_\infty^{g[j]}$  is called the *monoid of cofinite isometries of positive integers with the noise  $j$* . It was introduced in [4].

Any inverse semigroup  $S$  admits the *minimum group congruence*  $\mathfrak{C}_{\mathbf{mg}}$ :

$$a\mathfrak{C}_{\mathbf{mg}}b \text{ if and only if there exists } e \in E(S) \text{ such that } ea = eb.$$

**Proposition 1.** *Let  $\gamma$  and  $\delta$  be elements of the monoid  $\mathbf{IN}_\infty^{g[j]}$ . Then  $\gamma\mathfrak{C}_{\mathbf{mg}}\delta$  in  $\mathbf{IN}_\infty^{g[j]}$  if and only if  $n_\gamma^r - n_\gamma^d = n_\delta^r - n_\delta^d$ . Moreover, the quotient semigroup  $\mathbf{IN}_\infty^{g[j]}/\mathfrak{C}_{\mathbf{mg}}$  is isomorphic to the additive group of integers  $\mathbb{Z}(+)$  by the map*

$$\pi_{\mathfrak{C}_{\mathbf{mg}}}: \mathbf{IN}_\infty^{g[j]} \rightarrow \mathbb{Z}(+), \quad \gamma \mapsto n_\gamma^r - n_\gamma^d.$$

**Example 2.** We put  $\mathcal{C}\mathbf{IN}_\infty^{g[j]} = \mathbf{IN}_\infty^{g[j]} \sqcup \mathbb{Z}(+)$  and extend the multiplications from  $\mathbf{IN}_\infty^{g[j]}$  and  $\mathbb{Z}(+)$  onto  $\mathcal{C}\mathbf{IN}_\infty^{g[j]}$  in the following way:

$$k \cdot \gamma = \gamma \cdot k = k + (\gamma)\pi_{\mathfrak{C}_{\mathbf{mg}}} \in \mathbb{Z}(+), \quad \text{for all } k \in \mathbb{Z}(+) \text{ and } \gamma \in \mathbf{IN}_\infty^{g[j]}.$$

**Theorem 3.** *For any positive integer  $j$  every Hausdorff shift-continuous topology  $\tau$  on  $\mathbf{IN}_\infty^{g[j]}$  is discrete.*

**Proposition 4.** *Let  $j$  be any positive integer and  $\mathbf{IN}_\infty^{g[j]}$  be a proper dense subsemigroup of a Hausdorff semitopological semigroup  $S$ . Then  $I = S \setminus \mathbf{IN}_\infty^{g[j]}$  is a closed ideal of  $S$ .*

**Theorem 5.** *Let  $j$  be any positive integer and  $\mathbf{IN}_\infty^{g[j]}$  be a proper dense subsemigroup of a Hausdorff topological inverse semigroup  $S$ . Then  $I = S \setminus \mathbf{IN}_\infty^{g[j]}$  is a topological group.*

**Corollary 6.** *Let  $j$  be any positive integer and  $\mathbf{IN}_\infty^{g[j]}$  be a proper dense subsemigroup of a Hausdorff topological inverse semigroup  $S$ . Then the group  $S \setminus \mathbf{IN}_\infty^{g[j]}$  contains a dense cyclic subgroup.*

**Example 7.** Let  $\mathcal{C}\mathbf{IN}_\infty^{g[j]}$  be a semigroup defined in Example 2. Put  $M$  be an arbitrary subset of  $\{2, \dots, j\}$ . We define the topology  $\tau_{\mathbf{IC}}^M$  on  $\mathcal{C}\mathbf{IN}_\infty^{g[j]}$  in the following way:

- (i) all elements of the monoid  $\mathbf{IN}_\infty^{g[j]}$  are isolated points in  $(\mathcal{C}\mathbf{IN}_\infty^{g[j]}, \tau_{\mathbf{IC}}^M)$ ;
- (ii) for any  $k \in \mathbb{Z}(+)$  the family  $\mathcal{B}_{\mathbf{IC}}^M(k) = \{U_i^M(k) : i \in \mathbb{N}\}$ , where

$$U_i^M(k) = \{k\} \cup \{\gamma \in \mathcal{C}\mathbf{IN}_\infty^{g[j]}[M] : k \preceq \gamma \text{ and } n_\gamma^d \geq i\},$$

is the base of the topology  $\tau_{\mathbf{IC}}^M$  at the point  $k \in \mathbb{Z}(+)$ .

**Theorem 8.** *Let  $j$  be any positive integer and  $\mathbf{IN}_\infty^{g[j]}$  be a proper dense subsemigroup of a Hausdorff locally compact topological inverse semigroup  $(S, \tau)$ . Then  $(S, \tau)$  topologically isomorphic to the topological inverse semigroup  $(\mathcal{C}\mathbf{IN}_\infty^{g[j]}, \tau_{\mathbf{IC}}^M)$  for some subset  $M$  of  $\{2, \dots, j\}$ .*

**Corollary 9.** *For any positive integer  $j$  there exists  $(j-1)!+1$  distinct topologically non-isomorphic Hausdorff locally compact semigroup inverse topologies on the monoid  $\mathbf{CIN}_{\infty}^{[j]}$ .*

The obtained results generalize the corresponding results of the papers [1] and [3].

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