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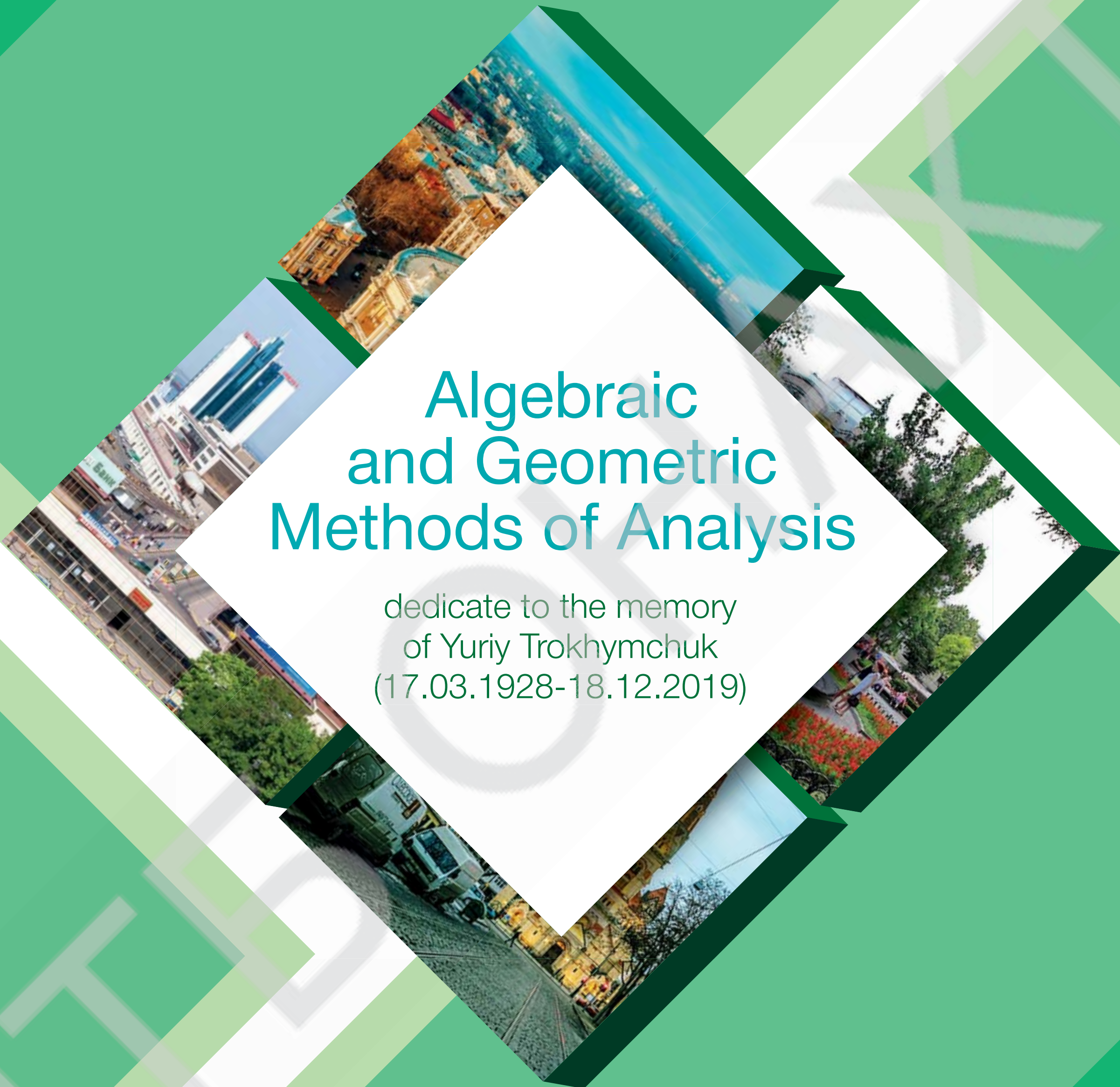

Algebraic and Geometric Methods of Analysis

dedicate to the memory
of Yuriy Trokhymchuk
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List of topics

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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The Tucker HO-SVD and the anisotropy of Finslerian geometric models

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The tensor spectral theory, as an extension of the classic spectral theory of linear operators reached sound applications in Big Data and Image Processing, based on the main decomposition tools of Tucker type, originating in the cornerstone HO-SVD decomposition [7, 8]. This theory enhances the statistical analysis in various fields, originally including MRI-imaging, Special Relativity, ecology, and HARDI biology. The present talk provides a brief survey of results recent tensor spectral theory, and of its applications to geometric structures which rely on *anisotropic metrics of Finsler type*. Several models are addressed, for which we derive the Tucker type HO-SVD decomposition and the induced powerful approximation provides aids for identifying main geometric features, and consistent anisotropy estimates for the Finslerian structures.

We include the mth candidate models for Special Relativity (Pavlov-Chernov, Bogoslovsky, and Roxbourgh models), for ecology (P.L. Antonelli &al.), HARDI biology (L. Astola & al.), Garner oncology and the physics of Langmuir-Blodgett monolayers.

We also present a brief survey of results from the spectral theory of covariant main symmetric tensors which rely on the fundamental tensor fields of the anisotropic geometric models. We note that the spectral data describe properties of the indicatrices associated to the Finsler norms, point out their asymptotic properties, and allow to derive best rank-I approximations - which provide simpler consistent estimates for the original anisotropic structures. We investigate the spectral data of covariant symmetric tensor fields, and focus on the metric and Cartan fields of the Finsler structures - including the Euclidean and Riemannian subcases - and further provide and discuss natural alternatives of the spectral equations.

We consider $n-$dimensional Finsler structures (M, F) with the main axioms relaxed by either dropping the positivity condition, or reducing the domain, and replacing the positive-definiteness of the Finsler metric d-tensor field with the non-degeneracy and constant signature condition. We shall denote by $g_{ij} = \frac{1}{2}\frac{\partial^2 F^2}{\partial y^i \partial y^j}$ and $C_{ijk} = \frac{1}{4}\frac{\partial^3 F^2}{\partial y^i \partial y^j \partial y^k}$ the components of the metric and Cartan d-tensor fields, respectively. One of the important features of the Cartan tensor is that its vanishing makes g quadratic in y, and consequently the Finsler space becomes Riemannian (correspondingly, pseudo-Finsler spaces become, in such case, pseudo-Riemannian).

For a real m-covariant symmetric tensor field T on the flat manifold $V = \mathbb{R}^n$ endowed with the Euclidean metric g, we say that a real λ is a Z-eigenvalue and that a vector y is an Z-eigenvector associated to λ, if they satisfy the system:

$$T.y^{m-1} = \lambda \cdot y, \quad g(y,y) = 1, \qquad \text{where} \quad T.y^{m-1} = \sum_{i,i_2,\dots,i_m \in \overline{1,n}} T_{i,i_2\dots i_m} y_{i_2} \cdot \ldots \cdot y_{i_m} dx^i,$$

where by lower dot is repeated transvection and the power is tensorial. As well, an alternative for spectral objects is the H-eigenvalue λ and its H-eigenvector, described by the homogeneous polynomial system: $(T.y^{m-1})_k = \lambda (y^k)^{m-1}$. Regarding the spectra consistency, it is known that in the Euclidean subcase, the Z- and the H-spectra are nonempty for even symmetric tensors, and that a symmetric tensor T is positive defynite/semi-definite iff all its $H-$ (or $Z-$) eigenvalues are positive/non-negative.

Among the applications of the spectral Finsler approach, there was observed the relevance of eigendata of the metric and Cartan tensors of the Langmuir-Finsler structure [1, 5, 9] within the physics of monolayers which studies the interphase boundary of a mono-molecular system.

For the Langmuir model, the Cartan tensor

$$C = C_{ijk}(x, y_*) dx^i \otimes dx^j \otimes dx^k, \quad C_{ijk}(x, y) = \frac{1}{4} \frac{\partial^3 F^2}{\partial y^i \partial y^j \partial y^k}, \tag{1}$$

considered at the fixed supporting element $(x, y_*) \in \widetilde{TM}$, has the 1-mode provided by the slices

$$C = (C_{1ij} = \gamma \cdot M, \;\; C_{2ij} = \nu \cdot M, \;\; C_{3ij} = O_{3\times 3}),$$

where $\gamma = \frac{3A}{\beta^3}$, $\nu = -\frac{\alpha}{\beta}\gamma$, $y_* = (\alpha, \beta, \gamma) \in T_x M$ is the supporting element, and $M = \begin{pmatrix} \beta^2 & -\alpha\beta & 0 \\ -\alpha\beta & \alpha^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

Theorem 1. *Consider the Cartan tensor of the Cartan-Langmuir tensor* (1) *from above. Then the associated spectral data are given as follows*

a) The Z-eigendata are given by:

$$S_{\lambda_1=0} = \left\{ \frac{1}{\sqrt{\alpha^2+\beta^2+t^2}} (\alpha, \beta, t) \,\middle|\, t \in \mathbb{R} \right\}, \quad S_{\lambda_2 = \frac{9A^2}{\beta^8 \sqrt{\alpha^4+\beta^4}}} = \left\{ \frac{\pm 1}{\sqrt{\alpha^4+\beta^4}} (\beta^2, \alpha^2, 0) \right\} \ni v_{\pm}.$$

b) The H-eigendata are the following:

$$S_{\lambda_1=0} = \left\{ \frac{1}{\sqrt{\alpha^2+\beta^2+t^2}} (\alpha, \beta, t) \,\middle|\, t \in \mathbb{R} \right\}, \quad S_{\lambda_2 = \frac{9A^2(\beta^2-\alpha^2)^2}{\beta^8}} = \left\{ \pm \left(\frac{\beta}{\sqrt{\alpha^2+\beta^2}}, \frac{-\alpha}{\sqrt{\alpha^2+\beta^2}}, 0 \right) \right\}.$$

Corollary 2. *The Candecomp approximation of the Langmuir-Cartan tensor* (1) *is twofold:*

$$C \sim A = \lambda_2 \cdot v_{\pm} \otimes v_{\pm} \otimes v_{\pm}, \quad v_{\pm} \in S_{\lambda_2}.$$

Moreover, the HOSVD decomposition and the partial/total ranks of the Cartan tensor are shown to be relevant in estimating the anisotropy level of the direction-dependent Finsler structure. We also note that while the Z-eigenproblem allows a globally covariant alternative, the H-eigenproblem exhibits a strongly local character.

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Зміст