



International
Scientific Conference



Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian
Academy of Sciences, the
first director of the Institute of
Mathematics of NAS of Ukraine

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- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Equiaffine immersions of codimension two with flat connection

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We consider the affine immersions by K. Nomizu, T. Sasaki [1], namely $f : (M^n, \nabla) \rightarrow \mathbb{R}^{n+2}$. For a transversal frame ξ_1, ξ_2 and tangent vector fields X, Y we have the affine analogues of Gauss and Weingarten decompositions, namely

$$D_X f_*(Y) = f_*(\nabla_X Y) + h^\alpha(X, Y)\xi_\alpha,$$

$$D_X \xi_\alpha = -f_*(S_\alpha X) + \tau_\alpha^\beta(X)\xi_\beta,$$

where h^α are components of the affine fundamental form, S_α are shape operators, τ_α^β are forms of transversal connection (with respect to ξ_1, ξ_2).

The *Weingarten mapping* $S_x : Q_x \times T_x(M^n) \rightarrow T_x(M^n)$ is defined [2] as follows: $(\xi, X) \mapsto S_\xi X$ at every point $x \in M^n$ (where $T_x(M^n)$ and Q_x are tangent and transversal distributions.)

For an affine immersion $f : (M^n, \nabla) \rightarrow \mathbb{R}^{n+2}$ with a transversal frame $\{\xi_1, \xi_2\}$, an *induced volume element* θ on M^n is defined [1, 3, 4] as follows:

$$\theta(X_1, \dots, X_n) = \det(f_*(X_1), \dots, f_*(X_n), \xi_1, \xi_2).$$

The transversal distribution Q with frame $\{\xi_1, \xi_2\}$ is called *equiaffine* if $\nabla_X \theta = 0$ for all $X \in T_x(M^n), x \in M^n$. For two-codimension affine immersion this condition is equivalent [4] to

$$\tau_1^1(X) + \tau_2^2(X) \equiv 0.$$

With an equiaffine transversal distribution Q we have an *equiaffine structure* (∇, θ) on M^n .

We will consider an affine immersion $f : (M^n, \nabla) \rightarrow \mathbb{R}^{n+2}$ with flat connection ∇ and equiaffine transversal distribution. Two-codimensional affine surfaces with different additional properties have been studied by many authors. Flat affine surfaces in \mathbb{R}^4 with flat normal connection were studied in [3]. The description of a parallel affine immersions $(M^n, \nabla) \rightarrow \mathbb{R}^{n+2}$ with flat connection in dependence on the rank of the Weingarten mapping were given in [2].

Let us remind that in general case (codimension k) the kernel and the image of the Weingarten mapping is defined by $\ker S = \bigcap_{\alpha=1}^k \ker S_\alpha$, $\text{im } S = \bigcup_{\alpha=1}^k \text{im } S_\alpha$. We say that Weingarten mapping is p -dimensional if $\text{rank } S := \dim \text{im } S = p$. It was proved [6] that for the immersion

$f : (M^n, \nabla) \rightarrow \mathbb{R}^{n+k}$ (for $k < n$) with maximal pointwise codimension and flat connection ∇ the following relations hold true:

- 1) $\dim \ker S \geq n - k$; 2) $\ker h \subseteq \ker S$; 3) $\dim \operatorname{im} S \leq k$;
- 4) if $\dim \operatorname{im} S = k$, then $\dim \ker S = n - k$ and $\ker h = \ker S$.

It was also proved [6] that the distribution \mathcal{S} of the kernels of Weingarten mapping is integrable on M^n and there exists a transversal distribution which is stationary along the leaves of the foliation \mathcal{FS} .

Since in the case of codimension two we have $\dim \operatorname{im} S \leq 2$, $\dim \ker S \geq n - 2$, so we have only three possible values for the dimension of $\operatorname{im} S$, namely 0, 1, 2. The most studied are affine immersions with zero and two-dimensional Weingarten mapping.

It is well known that an affine immersion with zero Weingarten mapping ($S \equiv 0$) has a flat connection and it is affinely equivalent to the graph of certain smooth map $F : M^n \rightarrow \mathbb{R}^2$ (see for example [5, 1, 6]), i. e.

$$f : (u^1, \dots, u^n) \mapsto (u^1, \dots, u^n, f^1(u^1, \dots, u^n), f^2(u^1, \dots, u^n)).$$

Obviously, a graph immersion is equiaffine.

According to the properties which were discussed in [6], in case $\dim \operatorname{im} S = 2$ we obtain $\ker h = \ker S$ and $\dim \ker h = n - 2$. Therefore such a submanifold is a submanifold of rank two (by the rank of Gaussian (Grassmann) mapping) [7]. Rank-two submanifold is a ruled submanifold with $(n - 2)$ -dimensional rulings over a two-dimensional base. In case this submanifold is a cylinder, its connection is determined by the connection of the cylinder base. In the general case the problem on its connection remains open.

We obtain a parametrization of a submanifold with one-dimensional Weingarten mapping and given properties. Such a submanifold is a peculiar "mix" of a graph and a ruled submanifold.

The main result. *Let $f : (M^n, \nabla) \rightarrow \mathbb{R}^{n+2}$ be an affine immersion with rank two affine fundamental form, equiaffine structure, flat connection ∇ , one-dimensional Weingarten then there exists three types of its parametrization:*

- (i) $\vec{r} = g(u^1, \dots, u^n) \vec{a}_1 + \int \vec{\varphi}(u^1) du^1 + \sum_{i=2}^n u^i \vec{a}_i$;
- (ii) $\vec{r} = (g(u^2, \dots, u^n) + u^1) \vec{a} + \int v(u^1) \vec{\eta}(u^1) du^1 + \sum_{i=2}^n u^i \int \lambda_i(u^1) \vec{\eta}(u^1) du^1$;
- (iii) $\vec{r} = (g(u^2, \dots, u^n) + u^1) \vec{\rho}(u^1) + \int (v(u^1) - u^1) \frac{d\vec{\rho}(u^1)}{du^1} du^1 + \sum_{i=2}^n u^i \int \lambda_i(u^1) \frac{d\vec{\rho}(u^1)}{du^1} du^1$.

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Some vanishing theorems of sufficient character about holomorphically projective mappings of Kahlerian spaces on the whole

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The generalized Bochner technique (see, for example, [1]) allows to broad to the noncompact but complete Kahlerian spaces some well-known theorems of holomorphically projective unique definability that have been proved previously only to the compact ones (see, for example, [2]). In particular, the next statements are true.

Theorem 1. *Complete connected Kahlerian C^r -spaces K^n ($n > 2$, $r > 3$) with positive definite Ricci form don't admit non-trivial (different from affine) holomorphically projective mappings on the whole.*

Corollary 2. *Complete connected Kahlerian C^r -spaces K^n ($n > 2$, $r > 3$) that have sign-definite metric form sign of which coincides with the sign of scalar curvature don't admit non-trivial (different from affine) holomorphically projective mappings on the whole.*

Corollary 3. *Complete connected Kahlerian C^r -spaces K^n ($n > 2$, $r > 3$) that have positively definite metric form and non-positively definite on the set of symmetric tensors b^{ij} form*

$$R_{\alpha\gamma\sigma\beta}b^{\alpha\beta}b^{\gamma\sigma}$$

don't admit non-trivial (different from affine) holomorphically projective mappings on the whole.

Examples of Kahlerian spaces of considered types are known. In particular, complete connected Kahlerian C^r -spaces K^n ($n > 2$, $r > 3$) of constant non-positive holomorphic curvature with positively definite metric form satisfies conditions of the both corollaries.

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