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of analysis»

Book of abstracts



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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
- History and methodology of teaching in mathematics

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НТБ ОНАФТ

# Orientations of trees and signed Markov graphs

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For every vertex map  $\sigma : V(X) \rightarrow V(X)$  on a finite tree  $X$  one can construct its *Markov graph*  $\Gamma(X, \sigma)$  which is a digraph that encodes  $\sigma$ -covering relation between edges in  $X$ . By definition,  $\Gamma(X, \sigma)$  has a vertex set equals the edge set  $E(X)$  of  $X$  and for every edge  $uv \in E(X)$  its out-neighbourhood in  $\Gamma(X, \sigma)$  equals the edge set  $E([\sigma(u), \sigma(v)]_X)$  of a unique shortest  $\sigma(u) - \sigma(v)$  path in  $X$ .

Let  $\tau : E(X) \rightarrow V(X)$  be an orientation of a tree  $X$ . For each non-constant map  $\sigma : V(X) \rightarrow V(X)$  the orientation  $\tau$  defines a map  $s_\tau : A(\Gamma(X, \sigma)) \rightarrow \{1, -1\}$  in such a way that  $s_\tau(e_1, e_2) = 1$ , if  $pr_{e_2}(\sigma(\tau(e_1))) = \tau(e_2)$  and  $s_\tau(e_1, e_2) = -1$ , otherwise. The pair  $\Gamma^\tau(X, \sigma) = (\Gamma(X, \sigma), s_\tau)$  is the *signed Markov graph* of  $\sigma$ . Denote by  $M_{\Gamma^\tau(X, \sigma)}$  the adjacency matrix of  $\Gamma^\tau(X, \sigma)$ . If  $\sigma$  is a constant map, then by definition  $M_{\Gamma^\tau(X, \sigma)}$  is a null matrix. It is well-known (see [1, 2]) that for any fixed orientation  $\tau$  of  $X$  the correspondence  $\sigma \rightarrow M_{\Gamma^\tau(X, \sigma)}$  establishes a homomorphism from the full transformation semigroup  $T_n$  to the matrix semigroup  $Mat_{n-1}(\mathbb{Z})$ . Note that this correspondence is “almost” injective in the sense that  $M_{\Gamma^\tau(X, \sigma_1)} = M_{\Gamma^\tau(X, \sigma_2)}$  if and only if  $\sigma_1 = \sigma_2$  or  $\sigma_1$  and  $\sigma_2$  are both constant maps.

**Theorem 1.** [3] *For any orientation  $\tau$  and a map  $\sigma$  the trace of  $M_{\Gamma^\tau(X, \sigma)}$  equals  $|\text{fix } \sigma| - 1$ , where  $\text{fix } \sigma$  is the set of  $\sigma$ -fixed points.*

A map  $\sigma$  is called  $\tau$ -positive provided  $s_\tau \equiv 1$ . Similarly, one can define  $\tau$ -negative maps. By definition constant maps are  $\tau$ -positive and  $\tau$ -negative for all orientations  $\tau$ .

**Proposition 2.** *A map  $\sigma$  is  $\tau$ -positive for all  $\tau$  if and only if  $\sigma$  is a projection on some connected set of vertices. Similarly,  $\sigma$  is  $\tau$ -negative for all  $\tau$  if and only if  $\sigma$  is constant.*

For a map  $\sigma$  an edge  $uv \in E(X)$  is called  $\sigma$ -positive ( $\sigma$ -negative) if  $pr_{uv}(\sigma(u)) = u$  and  $pr_{uv}(\sigma(v)) = v$  ( $pr_{uv}(\sigma(u)) = v$  and  $pr_{uv}(\sigma(v)) = u$ ). Denote by  $p(X, \sigma)$  and  $n(X, \sigma)$  the number of  $\sigma$ -positive and  $\sigma$ -negative edges in  $X$ , respectively.

**Proposition 3.** *If a map  $\sigma$  is  $\tau$ -positive ( $\tau$ -negative) for some  $\tau$ , then  $n(X, \sigma) = 0$  ( $p(X, \sigma) = 0$ ).*

A map  $\sigma$  is called *metric* if  $d_X(\sigma(u), \sigma(v)) \leq d_X(u, v)$  for all pairs of vertices  $u, v \in V(X)$ . It is easy to see that  $\sigma$  is metric if and only if  $[\sigma(u), \sigma(v)]_X \subset \sigma([u, v]_X)$  for all  $u, v \in V(X)$ . A map  $\sigma$  is called *linear* provided  $\sigma([u, v]_X) \subset [\sigma(u), \sigma(v)]_X$  for all  $u, v \in V(X)$ .

**Proposition 4.** *Let  $\sigma$  be a metric or a linear map. Then  $n(X, \sigma) \leq 1$ . Moreover, the equality  $n(X, \sigma) = 1$  implies  $p(X, \sigma) = 0$ .*

**Theorem 5.** *Let  $\sigma$  be a metric or a linear map. If  $n(X, \sigma) = 0$ , then there exists an orientation  $\tau$  such that  $\sigma$  is  $\tau$ -positive. Similarly, if  $n(X, \sigma) = 1$  (and thus  $p(X, \sigma) = 0$ ), then there is  $\tau$  such that  $\sigma$  is  $\tau$ -negative.*

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<b>Konovenko N., Lychagin V.</b> <i>On projective classes of rational functions</i>	<b>71</b>
<b>Kozerenko S.</b> <i>Orientations of trees and signed Markov graphs</i>	<b>73</b>
<b>Kuzmenko T.</b> <i>Constructive description of <math>G</math>-monogenic mappings in the algebra of complex quaternions</i>	<b>74</b>
<b>Lyubashenko V.</b> <i>Moyal and Rankin-Cohen deformations of algebras</i>	<b>76</b>
<b>Markitan V.</b> <i>Fractal properties of sets associated with Markov representation of real numbers defined by a double stochastic matrix</i>	<b>78</b>
<b>Matsumoto K.</b> <i>Warped product semi-slant submanifolds in locally conformal Kaehler manifolds</i>	<b>79</b>
<b>Mormul P.</b> <i>Weak and strong nilpotentizability in the monster towers hosting flag distributions</i>	<b>80</b>
<b>Mukhamadiev F. G.</b> <i>The local density and the local weak density of <math>N_7^{\mathcal{O}}</math>-kernel of a topological space <math>X</math> and superextensions</i>	<b>82</b>
<b>Muradoglu Z., Gunduz Aras C.</b> <i>A study for decision making problems by using interval soft sets</i>	<b>84</b>
<b>Muradov R. S.</b> <i>Archimedean copula functions and their some algebraic properties with applications</i>	<b>85</b>
<b>Obikhod T. V.</b> <i>BPS states of Fourfolds as candidates for Kaluza-Klein modes</i>	<b>87</b>
<b>Parasyuk I. O.</b> <i>Landau-type inequalities for curves on Riemannian manifolds</i>	<b>88</b>
<b>Prislyak A., Prus A.</b> <i>Morse-Smale flows on torus with hole</i>	<b>90</b>
<b>Reinov O.</b> <i>On nuclear operators with trace <math>V = 1</math> and <math>V^2 = 0</math></i>	<b>91</b>
<b>Sabitov I. Kh.</b> <i>Multiple roots of the volume polynomials for polyhedra</i>	<b>92</b>
<b>Samokhvalov S.</b> <i>Theory of gravity in the affine frame</i>	<b>93</b>
<b>Shamolin M. V.</b> <i>Integrable systems with dissipation on the tangent bundle of two-dimensional manifold</i>	<b>94</b>
<b>Turhan T., Ayyildiz N.</b> <i>On geometry of spatial kinematics in Lorentzian space</i>	<b>96</b>
<b>Turhan T., Ayyildiz N.</b> <i>A study on the integral invariants of a closed spacelike ruled surface</i>	<b>97</b>
<b>Vasilchenko A. N.</b> <i>Dual modules over Steenrod algebra 2</i>	<b>98</b>
<b>Vlasenko I.</b> <i>Topology of the basin of attraction of surface endomorphisms.</i>	<b>100</b>
<b>Voloshyna V.</b> <i>About some properties of functions determined as transformations from <math>W^n</math> to <math>W^m</math>-representation</i>	<b>101</b>
<b>Vyhivska L.</b> <i>On the problem of product of inner radii symmetric non-overlapping domains</i>	<b>103</b>
<b>Yildirim S., Ayyildiz N.</b> <i>A Study on Rectifying Curves in Semi-Euclidean Spaces</i>	<b>104</b>
<b>Арсеньева О. Е., Кириченко В. Ф., Суровцева Е. В.</b> <i>Эрмитова геометрия почти контактного метрического многообразия</i>	<b>105</b>