International
Scientific Conference

# Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

## List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Conformal equivalence of 3-webs

**Konovenko N.**
(ONTU, Odesa, Ukraine)
*E-mail:* ngkonovenko@gmail.com

Let 3-web $W_3 \langle \omega_1, w_2, \omega_3 \rangle$ defined in a domain $D$ on the conformal plane $(\mathbb{R}^2, g)$. We say that this 3-web is *regular* in $D$ if in this domain:

(1) The discriminant

$$\widetilde{\Delta} = -4I_2 + I_1^2 + 18I_1I_2 - 4I_1^3 - 27I_2^3$$

differs from zero.

(2) Invariants

$$I_1 = \frac{J_2}{J_1^2} \quad and \quad I_2 = \frac{J_3}{J_1^3}$$

are functionally independent in the domain, that is, the differential 2-form $\Omega = dI_1 \wedge dI_2 \neq 0$.

Moreover, invariants $I_1, I_2$ are coordinates in the domain.

We remark that the elementary symmetric functions

$$\begin{aligned} J_1 &= \lambda_1 + \lambda_2 + \lambda_3, \\ J_2 &= \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3, \\ J_3 &= \lambda_1\lambda_2\lambda_3 \end{aligned}$$

are $S_3$ - invariants and $\lambda_1, \lambda_2, \lambda_3$ are positive smooth functions.

Let's number now forms $\omega_1, w_2, \omega_3$ in the domain and say that the 3-web is *oriented* in the domain if in this numbering $\omega_1 \wedge \omega_2 = r_{12}\Omega$, where $r_{12} > 0$. In this case we'll scale forms $\omega_i$ in such a way, that

$$\omega_1 \wedge \omega_2 = \Omega.$$

In opposite case, we call the 3-web *non-oriented* and scale the 1-forms $\omega_i$ in such a way, that

$$\omega_1 \wedge \omega_2 = -\Omega.$$

In these both cases we decompose 1-forms $\omega_i$ in the invariant coordinates $I_1, I_2$

$$\omega_i = \sum_{j=1}^{2} w_{ij} dI_j,$$

Then, all functions $w_{ij}, i = 1,2,3; j = 1,2$ are conformal invariants, satisfying the following additional relations

$$\sum_{i=1}^{3} w_{ij} = 0, \ \ j = 1,2.$$

Now, let's write down the standard metric tensor $g$ in invariant coordinates as follows

$$g = \sum_{i,j=1}^{2} g_{ij} dI_i \otimes dI_j.$$

Remark, that the volume 2-form $\Omega_g$, associated with metric $g$, is the following

$$\Omega_g = \sqrt{\det \|g_{ij}\|} dI_1 \wedge dI_2.$$

Therefore, the metric tensor

$$\widetilde{g} = \frac{g}{\sqrt{\det \|g_{ij}\|}}$$

has the associated volume form $\Omega_{\widetilde{g}} = \Omega$.

Finally, we get the following result.

**Theorem 1.** *Let 3-web $W_3 \langle \omega_1, w_2, \omega_3 \rangle$ be regular in a domain $D$ in the conformal plane $(\mathbb{R}^2, g)$. Then the above functions*

$$w_{ij}, \qquad \widetilde{g}_{ij} = \frac{g_{ij}}{\sqrt{\det \|g_{ij}\|}}$$

*that are components of 1-forms $\omega_1, \omega_2$ and the metric tensor $\widetilde{g}$ in the invariant coordinates $I_1, I_2$, are conformal invariants of plane 3-webs.*

*Moreover, any two regular 3-webs are conformly equivalent if and only if the corresponding functions $w_{ij}$ and $\widetilde{g}_{ij}$ coinside.*

# The fundamental group of Riemann surface via Riemann's existence theorem

**Yaacov Kopeliovich**
(University of Connecticut School of Business Storrs)
*E-mail:* yaacov.kopeliovich@uconn.edu

One of the classical things we learn in any complex analysis course is the structure of the fundamental group of Riemann surfaces that it is given by the following theorem:

**Theorem 1.** *The fundamental group of Riemann Surfaces of genus $g$ is given by $2g$ generators with one relation :*

$$\prod_{i=1}^{g} [a_i, b_i] = 1 \qquad (1)$$