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# Algebraic and Geometric Methods of Analysis

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## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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ІНСТИТУТ ОНАФТ

## Formal groups and algebraic cobordism

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We present our algorithm for constructing of Lazard's one dimensional universal commutative formal group. We apply it to some constructions of complex and algebraic cobordism.

### ALGORITHM FOR CONSTRUCTING OF LAZARD'S ONE DIMENSIONAL UNIVERSAL FORMAL GROUP

Here we follow to [1, 5, 8]. We extract from Lazard [1] an algorithm of constructing of Lazard's one dimensional universal formal group law. The constructions of  $n(n \geq 1)$ -buds, formal groupoids and moduli spaces of one dimensional formal groups are investigated and are used. Let  $A_q$  and  $A'_q$  be the rings of polynomials  $A_q = \mathbb{Z}[\alpha_1, \dots, \alpha_q]$  and  $A'_q = \mathbb{Q}[\alpha_1, \dots, \alpha_q]$ .

**Proposition 1.** *The structure of the algorithm has the next form. For the (one-dimensional) 1-bud  $x + y + \alpha_1 xy$  we put:*

$$f_1(x, y) = x + y + \alpha_1 xy; \quad \varphi_1 = x - \frac{1}{2}\alpha_1 x^2;$$

$$f_{q+1}(x, y) = f_q(x, y) + h'(x, y) + \alpha_{q+1} C_{q+2}(x, y), \quad f_q(x, y) \in A_q;$$

compute  $\varphi_{q+1}$ ,  $\varphi_{q+1} \in A'_q$ .

**Remark 2.** The algorithm and expressions for  $\varphi_{q+1}$ ,  $h'(x, y)$ ,  $C_{q+2}(x, y)$  will be given in my talk.

**Definition 3.** The ring  $\mathbb{L} = \mathbb{Z}[\alpha_1, \alpha_2, \dots]$  is called the Lazard ring.

### FORMAL GROUPS IN COMPLEX COBORDISM

Here we follow to [2, 3, 4, 6].

Let  $M$  be a smooth manifold,  $TM$  be the tangent bundle on  $M$  and  $\mathbb{R}^m$  the trivial real  $m$ -dimensional bundle of  $M$ .

**Definition 4.** A manifold  $M$  is *stably complex* if for some natural  $m$  the real vector bundle  $TM \oplus \mathbb{R}^m$  admits a complex structure.

Let  $M_1$  and  $M_2$  be two smooth manifolds of dimension  $n$ , and  $W$  be the smooth manifold of dimension  $n + 1$  with a boundary that is the union of  $M_1$  and  $M_2$ , i.e.  $\partial W = M_1 + M_2$ .

**Definition 5.** Let  $M_1, M_2, W$  be stably complex manifolds. In above notations the *complex (unitary) cobordism* between  $M_1$  and  $M_2$  is a manifold  $W$  whose boundary is the disjoint union of  $M_1, M_2$ ,  $\partial W = M_1 + \overline{M_2}$  where the corresponding structure on  $\partial W$  is induced from  $W$  and  $\overline{M_2}$  denotes the manifold with opposite structure.

**Remark 6.** Suppose we have the relation of complex cobordism. Then the relation divides stably complex manifolds on equivalence classes called classes of complex cobordisms.

**Lemma 7.** *The set of classes of complex cobordisms with operations of disjoint union and product of stable manifolds form commutative graded ring  $\mathbb{L} = \mathbb{Z}[v_1, v_2, \dots]$ .*

**Theorem 8** (A. S. Mishchenko). *Let  $g(t)$  be the logarithm of Lazard universal formal group, and  $[\mathbb{C}P^n]$  are classes of unitary cobordisms of complex projective spaces. Then*

$$g(t) = \sum_{n \geq 0} \frac{[\mathbb{C}P^n]}{n+1} t^{n+1}, \quad [\mathbb{C}P^0] = 1.$$

#### FORMAL GROUPS IN ALGEBRAIC COBORDISM

Here we follow to [7]. Let  $k$  be a field of characteristic zero and  $\mathbf{Sm}(k)$  be the full subcategory of smooth quasi-projective  $k$ -schemes of the category of separable finite-type  $k$ -schemes. Let  $A^*$  be an oriented cohomology theory on  $\mathbf{Sm}(k)$  and let  $c_1(L)$  be the first Chern class of line bundle  $L$  on  $X \in \mathbf{Sm}(k)$ .

**Proposition 9** (Quillen, [7]). *Let  $L, M$  be line bundles on  $X \in \mathbf{Sm}(k)$ . There exists formal group law  $F_A$  with coefficients in  $A^*$  such that*

$$c_1(L \otimes M) = F_A(c_1(L), c_1(M)).$$

**Theorem 10** (Levine-Morel). *There is a universal oriented cohomology theory  $\Omega$  over  $k$  called algebraic cobordism. The classifying map  $\phi_\Omega : \mathbb{L} \rightarrow \Omega^*(k)$  is an isomorphism, so  $F_\Omega$  is the universal formal group law.*

**Problem 11.** It seems that very little is known about applying  $n(n \geq 2)$ -dimensional commutative formal groups to cobordism theory. For instance what is the application of a two-dimensional 1-bud ?

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