International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of

Dvytro Grave

(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

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List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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In these both cases we decompose 1-forms ω_i in the invariant coordinates I_1, I_2

$$\omega_i = \sum_{j=1}^{2} w_{ij} dI_j,$$

Then, all functions $w_{ij}, i = 1,2,3; j = 1,2$ are conformal invariants, satisfying the following additional relations

$$\sum_{i=1}^{3} w_{ij} = 0, \; j = 1,2.$$

Now, let's write down the standard metric tensor g in invariant coordinates as follows

$$g = \sum_{i,j=1}^{2} g_{ij} dI_i \otimes dI_j.$$

Remark, that the volume 2-form Ω_g, associated with metric g, is the following

$$\Omega_g = \sqrt{\det \|g_{ij}\|} dI_1 \wedge dI_2.$$

Therefore, the metric tensor

$$\widetilde{g} = \frac{g}{\sqrt{\det \|g_{ij}\|}}$$

has the associated volume form $\Omega_{\widetilde{g}} = \Omega$.

Finally, we get the following result.

Theorem 1. *Let 3-web $W_3 \langle \omega_1, w_2, \omega_3 \rangle$ be regular in a domain D in the conformal plane $(\mathbb{R}^2, g)$. Then the above functions*

$$w_{ij}, \qquad \widetilde{g}_{ij} = \frac{g_{ij}}{\sqrt{\det \|g_{ij}\|}}$$

that are components of 1-forms ω_1, ω_2 and the metric tensor $\widetilde{g}$ in the invariant coordinates I_1, I_2, are conformal invariants of plane 3-webs.

Moreover, any two regular 3-webs are conformly equivalent if and only if the corresponding functions w_{ij} and $\widetilde{g}_{ij}$ coinside.

The fundamental group of Riemann surface via Riemann's existence theorem

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One of the classical things we learn in any complex analysis course is the structure of the fundamental group of Riemann surfaces that it is given by the following theorem:

Theorem 1. *The fundamental group of Riemann Surfaces of genus g is given by $2g$ generators with one relation :*

$$\prod_{i=1}^{g} [a_i, b_i] = 1 \tag{1}$$

$[a_i, b_i]$ *is the commutator of* 2 *group elements given by:* $[x, y] = xy(yx)^{-1}$

However when you first encounter Algebraic curves (Riemann Surfaces) they are presented through cuts and analytic continuation in a pictersque way. I have never seen a proof in the literature that the fundamental group of the surface given pictorially by cuts has a representation given by the theorem. Indeed the starting point of surface groups is the commutation relation. In this talk I will try to fill this gap. While I don't have a formal proof yet I will present some results that to me seems somewhat surprising. The talk is elementary in nature and no knowledge of heavy topology is required.

References

[1] Mike Fried. Combinatorial Computation of Moduli Dimension of Nielsen Classes of Covers Emphasis on the solvable cover case with historical comments from Zariski 1989 Contemporary Mathematics .

Problem with integral conditions for evolution equations in Banach space

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Let A be a given linear operator acting in the Banach space B, and for this operator, arbitrary powers $A^n : B \to B$, $n \in \mathbb{N}$. Denote be $x(\lambda)$ the eigenvector of the operator A which corresponds to its eigenvalue $\lambda \in \Lambda$, i.s. nonzero solution in B of the equation $Ax(\lambda) = \lambda x(\lambda), \lambda \in \Lambda$, where $\lambda \subset \mathbb{C}$. If Λ is not an eigenvalue of the operator A then $x(\lambda) = 0$.

We consider next problem with integrals condition

$$\frac{d^2U}{dt^2} + a(A)\frac{dU}{dt} + b(A)U = 0, \quad t \in [0, T], \tag{1}$$

$$\int_0^T U(t)dt = \varphi_1, \quad \int_0^T tU(t)dt = \varphi_2, \tag{2}$$

where $\varphi_1, \varphi_2 \in B$, $T > 0$, $u : (0; \alpha) \cup (\beta; h) \to B$ - is an unknown function, $a(A) : B \to B$, $b(A) : B \to B$ - is abstract operators with entire symbols $a(\lambda) \neq const$, $b(\lambda) \neq const$.

Let for $m = \{0, 1\}$ function $M_m(t, \lambda)$ be a solution of the problem

$$\frac{d^2M_m(t,\lambda)}{dt^2} + a(\lambda)\frac{dM_m(t,\lambda)}{dt} + b(\lambda)M_m(t,\lambda) = 0, \quad t \in [0, T], \tag{3}$$

$$\int_0^T t^k M_m(t,\lambda)dt = \delta_{km}, \quad k = \{0, 1\}, \tag{4}$$

where δ_{km} is the Kronecker symbol.

Definition. We shall say that vectors $\varphi_1, \varphi_2 \in B$, from B belong $L \subset B$. If dependent exists on linear operators $R_{\varphi_k}(\lambda) : B \to B$, $\lambda \in \Lambda$ and measures μ_{φ_k} such that

$$\varphi_k = \int_\Lambda R_{\varphi_k}(\lambda)x(\lambda)d\mu_{\varphi_k(\lambda)}. \tag{5}$$