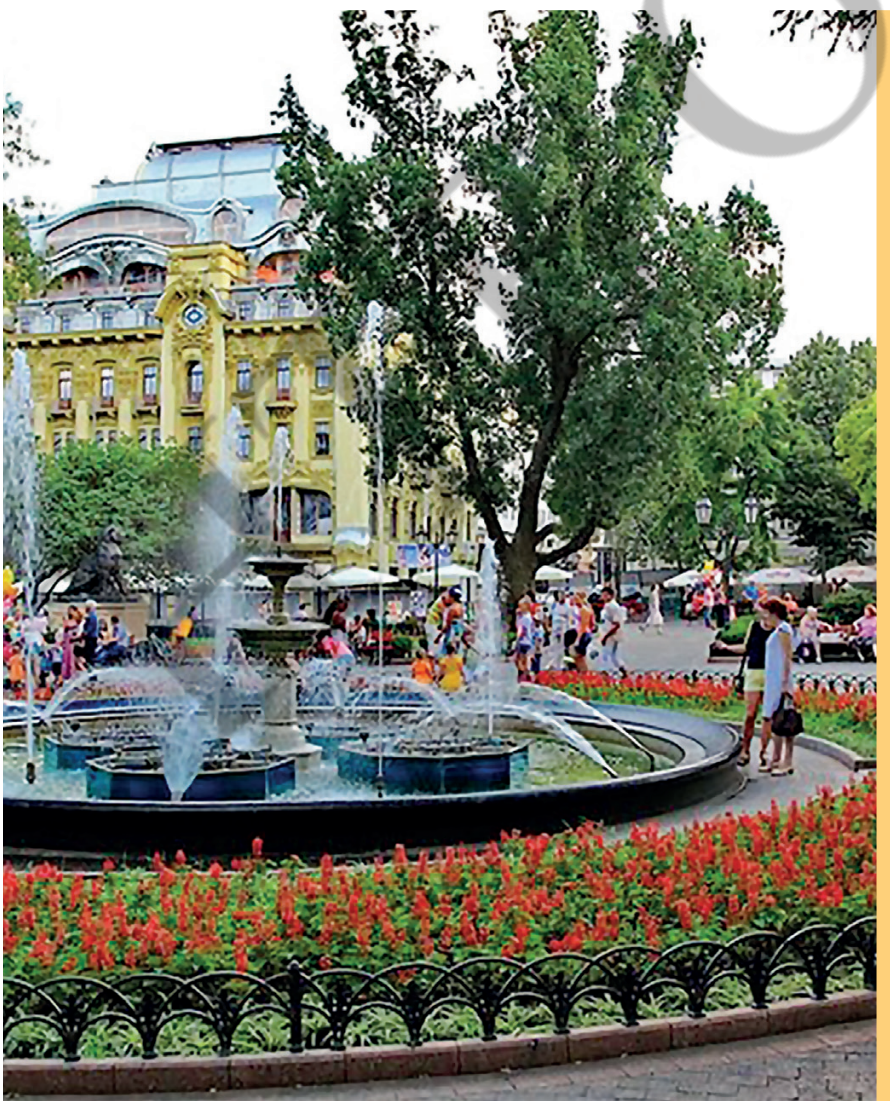
International
Scientific Conference

Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
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- Kyiv Mathematical Society

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About some Steiner trees

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I will talk about the famous Steiner problem.

We denote by $\mathcal{H}^1$ the linear Haurdorff measure (roughly speaking, length).

Problem 1 (Euclidean Steiner problem)**.** Let C be a compact subset of $\mathbb{R}^d$. To find a closed S such that $S \cup C$ is connected and $\mathcal{H}^1(S)$ is minimal.

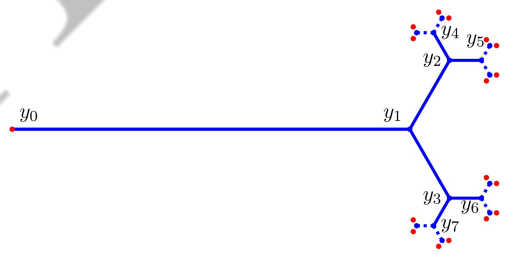
Some properties (if $\mathcal{H}^1(S) < \infty$) hold:

- S exists;
- S contains no loops;
- only two variants of neighbourhoods for points from $S \setminus C$;
- only two variants of a neighbourhood of a point $x \in S \setminus C$:
  - a regular tripod (x is a **branching point**);
  - a segment; x is an inner point.
- S contains at most countable number of branching points.

Usually S is usually called **Steiner tree**, and it is called **indecomposable** (irreducible, full), when $S \setminus C$ is connected. If C is totally disconnected then S should be connected.

Theorem 2 (Paolini–Stepanov–T, 2015; Cherkashin–T. 2023; Paolini–Stepanov 2023)**.** *There is a compact planar set C for which the unique solution of the Steiner problem is indecomposable and has infinite number of triple points.*

In the theorem C and Σ are self-similar fractals with a sufficiently small scale factor.



Theorem 3 (Basok, Cherkashin, T., 2022)**.** *In the plane for $m \geq 4$ the set of m terminals (considered as a subset of $\mathbb{R}^{2m}$) with non unique Steiner trees has the Hausdorff dimension $2m - 1$.*

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The multiplicities of non-acyclic SL_2-representations and L-functions of twisted Whitehead links

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We briefly survey a joint work [2] with Léo Bénard, Ryoto Tange, and Anh T. Tran, which is a continuation of our previous work [8] (See also [6, 9, 1, 7] and [4, 5, 3]).

We consider a natural divisor on $\mathrm{SL}_2\mathbb{C}$-character varieties of knots and links, given by the so-called acyclic Reidemeister torsion. We provide a geometric interpretation of this divisor. We focus on the particular family of odd twisted Whitehead links W_{2k-1}, where we show that this divisor has multiplicity two. Moreover, we apply these results to the study of the L-functions of the universal deformations of representations over finite fields of twisted Whitehead links.

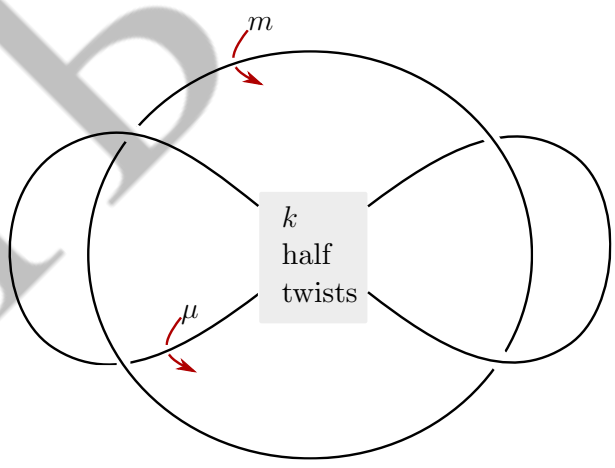

FIGURE 0.1. The twisted Whitehead link W_k