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# Asymptotic behavior of solutions to a nonlinear Beltrami equation

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Let  $D$  be a domain in  $\mathbb{C}$  and  $\mu: D \rightarrow \mathbb{C}$  be a measurable function with  $|\mu(z)| < 1$  a.e. in  $D$ . The linear PDE

$$f_{\bar{z}} = \mu(z)f_z. \quad (1)$$

is called the Beltrami equation; here  $z = x + iy$ ,

$$f_{\bar{z}} = \frac{1}{2}(f_x + if_y), \quad f_z = \frac{1}{2}(f_x - if_y).$$

The function  $\mu$  is called the complex dilatation.

Let  $\sigma: D \rightarrow \mathbb{C}$  be a measurable function, and  $m \geq 0$ . We consider the following nonlinear equation

$$f_r = \sigma(re^{i\theta})|f_\theta|^m f_\theta, \quad (2)$$

written in the polar coordinates  $(r, \theta)$ . Here  $f_r$  and  $f_\theta$  are the partial derivatives of  $f$  in  $r$  and  $\theta$ , respectively, satisfying

$$rf_r = zf_z + \bar{z}f_{\bar{z}}, \quad f_\theta = i(zf_z - \bar{z}f_{\bar{z}}).$$

The equation (2) in the Cartesian coordinates has the form

$$f_{\bar{z}} = \frac{A(z)|zf_z - \bar{z}f_{\bar{z}}|^m - 1}{A(z)|zf_z - \bar{z}f_{\bar{z}}|^m + 1} \frac{z}{\bar{z}} f_z, \quad (3)$$

where  $A(z) = \sigma(z)|z|i$ .

Note that in the case  $m = 0$ , the equation (2) is the usual Beltrami equation (1) with the complex dilatation

$$\mu(z) = \frac{z}{\bar{z}} \frac{A(z) - 1}{A(z) + 1}.$$

Let  $\mathbb{B} = \{z \in \mathbb{C} : |z| < 1\}$ .

**Theorem 1.** *Let  $f: \mathbb{B} \rightarrow \mathbb{B}$  be a regular homeomorphic solution of the equation (2) which belongs to Sobolev class  $W_{\text{loc}}^{1,2}$ , and normalized by  $f(0) = 0$ . Assume that the coefficient  $\sigma: \mathbb{B} \rightarrow \mathbb{C}$  satisfies the following condition*

$$\liminf_{r \rightarrow 0} \left( \frac{1}{\pi r^2} \iint_{|z| < r} \frac{dxdy}{|z| (\text{Im } \bar{\sigma}(z))^{\frac{1}{m+1}}} \right)^{m+1} \leq \sigma_0 < \infty.$$

Then

$$\liminf_{z \rightarrow 0} \frac{|f(z)|}{|z|} \leq c_m \sigma_0^{\frac{1}{m}} < \infty,$$

where  $c_m$  is a positive constant depending on the parameter  $m$ .

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