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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

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ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБОТИ

Inverse problem for tree of Stieltjes strings

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Finite-dimensional spectral problems on an interval were considered in [1] (some recent results see in [4], [3] and applications in [2]). Finite-dimensional spectral problems on graphs occur in various fields of physics (see e.g. [5], [6] and [7]).

We consider a tree T rooted at a pendant vertex. All edges are directed away from the root. Each edge e_j of this tree is a Stieltjes string with point masses m_k^j ($k = 1, 2, \dots, n_j$, $j = 1, 2, \dots, q$) and subintervals l_k^j ($k = 0, 1, \dots, n_j$). The total length of e_j is $l_j = \sum_{k=0}^{n_j} l_k^j$. We denote $\tilde{n}_j = n_j - 1$ if $l_{n_j}^j = 0$ and $\tilde{n}_j = n_j$ if $l_{n_j}^j > 0$. The Dirichlet problem on this tree consists of the following equations.

For each edge:

$$\frac{u_k^j - u_{k+1}^j}{l_k^j} + \frac{u_k^j - u_{k-1}^j}{l_{k-1}^j} - m_k^j \lambda^2 u_k^j = 0, \quad (k = 1, 2, \dots, \tilde{n}_j, j = 1, 2, \dots, q). \quad (1)$$

For each interior vertex with incoming edge e_j and outgoing edges e_r we have

$$u_{\tilde{n}_j+1}^j = u_0^r, \quad (2)$$

and

$$\frac{u_{\tilde{n}_j+1}^j - u_{\tilde{n}_j}^j}{l_{\tilde{n}_j}^j} + \sum_r \frac{u_0^r - u_1^r}{l_0^r} = \begin{cases} 0, & \text{if } l_{\tilde{n}_j}^j > 0, \\ -m_{\tilde{n}_j}^j \lambda^2 u_{\tilde{n}_j}^j, & \text{if } l_{\tilde{n}_j}^j = 0. \end{cases} \quad (3)$$

For an edge e_j incident with a pendant vertex (except of the root) we have the Dirichlet boundary condition:

$$u_{n_j+1}^j = 0. \quad (4)$$

If e_1 is the edge incident with the root then at the root we have

$$u_0^1 = 0. \quad (5)$$

The Neumann problem consists of equations (1)–(4) and

$$u_0^1 = u_1^1. \quad (6)$$

at the root.

First of all we notice that interior vertices of degree 2 do not influence the results and we can assume absence of such vertices without losses of generality. Let P be a path in the tree T involving the maximum number of masses. Obviously it starts and finishes with pendant vertices. We denote the initial vertex of P by v_0 and choose it as the root of the tree. The enumeration of other vertices is arbitrary. We denote the edge incoming into a vertex v_i by e_i for all i . Then $P : v_0 \rightarrow v_1 \rightarrow v_{s_2} \rightarrow v_{s_3} \rightarrow \dots \rightarrow v_{s_{r-1}} \rightarrow v_{s_r}$. Deleting v_0 and e_1 we obtain a new tree T' rooted at the vertex v_1 .

Since the degree of v_1 is $d(v_1) > 2$ we can divide our tree T' into subtrees $T'_1, T'_2, \dots, T'_{d(v_1)-1}$ having v_1 as the only common vertex. (We say that $T'_1, T'_2, \dots, T'_{d(v_1)-1}$ are *complementary subtrees* of T' .)

Denote by $\phi_{N(v_0)}(z)$ (where $z = \lambda^2$) the characteristic polynomial of problem (1)– (4), (6) on the tree T and by $\phi_{D(v_0)}(z)$ the characteristic polynomial of problem (1)– (4), (5) on this tree. These polynomials are normalized such that

$$\frac{\phi_{D(v_0)}(0)}{\phi_{N(v_0)}(0)} = l_1 + \frac{1}{\sum_{r=1}^{d(v_1)-1} \frac{\phi_{N_r(v_1)}(0)}{\phi_{D_r(v_1)}(0)}},$$

$\phi_{D,r(v_1)}(z)$ is the characteristic polynomial of the Dirichlet problem (1)– (4), (5) on T'_r and $\phi_{N,r(v_1)}(z)$ is the characteristic polynomial of the Neumann problem (1)– (4), (6) on T'_r and so on.

The inverse problem lies in recovering the spectral problem data $\{m_k^j\}_{k=1}^{n_j}$, $\{l_k^j\}_{k=0}^{n_j}$ using the spectra $\{\mu_k\}_{k=-n, k \neq 0}^n$ and $\{\nu_k\}_{k=-n, k \neq 0}^n$ of the Neumann and Dirichlet problems.

The following theorem gives sufficient conditions for existence of solution of such inverse problem.

Theorem 1. Let $\{\mu_k\}_{k=-n, k \neq 0}^n$ and $\{\nu_k\}_{k=-n, k \neq 0}^n$ be symmetric ($\mu_{-k} = -\mu_k$, $\nu_{-k} = -\nu_k$) and monotonic sequences of real numbers which interlace:

$$0 < (\mu_1)^2 < (\nu_1)^2 < \dots < (\mu_n)^2 < (\nu_n)^2.$$

Let T be a metric tree of a prescribed form rooted at a pendant vertex v_0 with prescribed lengths of edges $l_j > 0$ ($j = 1, 2, \dots, q$, q is the number of edges in T).

Then

1) there exist numbers $n_j \in \{0\} \cup \mathbb{N}$ ($j = 1, 2, \dots, q$), sequences of positive numbers $\{m_k^j\}_{k=1}^{n_j}$ (point masses on the edge e_j , $j = 1, 2, \dots, q$) and numbers $\{l_k^j\}_{k=0}^{n_j}$ ($l_k^j > 0$ for all $k = 0, 1, \dots, n_j - 1$, $l_{n_j} \geq 0$ for all $j = 1, 2, \dots, q$ such that $\sum_{k=0}^{n_j} l_k^j = l_j$, $\sum_{j=1}^q n_j = n$, the spectrum of Neumann problem (1)–(4), (6), coincides with $\{\mu_k\}_{k=-n, k \neq 0}^n$ and the spectrum of Dirichlet problem (1)–(4), (5) coincides with $\{\nu_k\}_{k=-n, k \neq 0}^n$;

2) the two spectra $\{\mu_k\}_{k=-n, k \neq 0}^n$ and $\{\nu_k\}_{k=-n, k \neq 0}^n$ and the length l_1 of the edge incident with the root uniquely determine the masses $\{m_k^1\}_{k=1}^{n_1}$ (point masses on the edge e_1) and lengths $\{l_k^1\}_{k=0}^{n_1}$ of the subintervals on this edge.

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