

International
Scientific Conference



Algebraic
and Geometric
Methods
of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

SCIENTIFIC COMMITTEE

- | | |
|---|--|
| • Vladimir Balan (<i>Bucharest, Romania</i>) | • Volodymyr Lyubashenko (<i>Kyiv, Ukraine</i>) |
| • Taras Banakh (<i>Lviv, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Dmytro Bolotov (<i>Kharkiv, Ukraine</i>) | • Koji Matsumoto (<i>Yamagata, Japan</i>) |
| • Vyacheslav Boyko (<i>Kyiv, Ukraine</i>) | • Piotr Mormul (<i>Warsaw, Poland</i>) |
| • Yulia Fedchenko (<i>Odesa, Ukraine</i>) | • Maryna Nesterenko (<i>Kyiv, Ukraine</i>) |
| • Oleg Gutik (<i>Lviv, Ukraine</i>) | • Roman Popovych (<i>Kyiv, Ukraine</i>) |
| • Olena Karlova (<i>Chernivtsi, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |
| • Volodymyr Kiosak (<i>Odesa, Ukraine</i>) | • Aleksandr Savchenko (<i>Kherson, Ukraine</i>) |
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | |

ORGANIZING COMMITTEE

- | | |
|---|--|
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | • Bohdan Mazhar (<i>Kyiv, Ukraine</i>) |
| • Yuliya Fedchenko (<i>Odesa, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Mykola Lysynskiy (<i>Kyiv, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |

where

$$\begin{aligned}
C_1 &= \beta \sqrt{\frac{\sqrt{78+6\sqrt{5}}-8}{2}}, & D_1 &= \alpha \sqrt{\frac{\sqrt{78-6\sqrt{5}}-8}{2}}, \\
C_2 &= \frac{\sqrt{\sqrt{78+6\sqrt{5}}-8} \sqrt{\sqrt{78-6\sqrt{5}}}}{-5+\sqrt{5}+\sqrt{78+6\sqrt{5}}}, & D_2 &= \frac{\sqrt{\sqrt{78-6\sqrt{5}}-8} \sqrt{\sqrt{78+6\sqrt{5}}}}{5+\sqrt{5}-\sqrt{78-6\sqrt{5}}}, \\
C_3 &= \frac{(148-112\sqrt{5}-\sqrt{78+6\sqrt{5}}(25-11\sqrt{5}))\sqrt{(78-6\sqrt{5})^3}}{(-5+\sqrt{5}+\sqrt{78+6\sqrt{5}})^2\sqrt{\sqrt{78+6\sqrt{5}}-8}}, \\
D_3 &= \frac{(148+112\sqrt{5}-\sqrt{78-6\sqrt{5}}(25+11\sqrt{5}))\sqrt{(78+6\sqrt{5})^3}}{(5+\sqrt{5}-\sqrt{78+6\sqrt{5}})^2\sqrt{\sqrt{78-6\sqrt{5}}-8}}.
\end{aligned}$$

Note that since $\binom{2n}{n} = (n+1)C_n$, where C_n are Catalan numbers, our results could be stated equivalently in terms of the Catalan numbers. Similar series were studied recently in [2, 3, 4, 5].

REFERENCES

- [1] K. Adegoke, R. Frontczak and T. Goy. Evaluation of some alternating series involving the binomial coefficients $C(4n, 2n)$. Preprint arXiv:2404.05770v1 [math.NT], 2024.
- [2] K. Adegoke, R. Frontczak and T. Goy. Fibonacci–Catalan series. *Integers*, 22: #A110, 2022.
- [3] K. Adegoke, R. Frontczak and T. Goy. On a family of infinite series with reciprocal Catalan numbers. *Axioms*, 11: Art. 165, 2022.
- [4] N. Bhandari. Infinite series associated with the ratio and product of central binomial coefficients. *Journal of Integer Sequences*, 25: Art. 22.6.5, 2022.
- [5] H. Chen. Interesting Ramanujan-like series associated with powers of central binomial coefficients. *Journal of Integer Sequences*, 25: Art. 22.1.8, 2022.

On non-topologizable semigroups

Oleg Gutik

(Ivan Franko National University of Lviv, Universytetska 1, Lviv, 79000, Ukraine)

E-mail: oleg.gutik@lnu.edu.ua, ogutik@gmail.com

In this paper we shall follow the semigroup terminology of [1, 2, 3, 4].

Throughout these abstract we always assume that all topological spaces involved are Hausdorff — unless explicitly stated otherwise.

Definition 1. Let X, Y and Z be topological spaces. A map $f: X \times Y \rightarrow Z$, $(x, y) \mapsto f(x, y)$, is called

- (i) *right [left] continuous* if it is continuous in the right [left] variable; i.e., for every fixed $x_0 \in X$ [$y_0 \in Y$] the map $Y \rightarrow Z$, $y \mapsto f(x_0, y)$ [$X \rightarrow Z$, $x \mapsto f(x, y_0)$] is continuous;
- (ii) *separately continuous* if it is both left and right continuous;
- (iii) *jointly continuous* if it is continuous as a map between the product space $X \times Y$ and the space Z .

Definition 2. Let S be a non-void topological space which is provided with an associative multiplication (a semigroup operation) $\mu: S \times S \rightarrow S$, $(x, y) \mapsto \mu(x, y) = xy$. Then the pair (S, μ) is called

- (i) a *right topological semigroup* if the map μ is right continuous, i.e., all interior left shifts $\lambda_s: S \rightarrow S, x \mapsto sx$, are continuous maps, $s \in S$;
- (ii) a *left topological semigroup* if the map μ is left continuous, i.e., all interior right shifts $\rho_s: S \rightarrow S, x \mapsto xs$, are continuous maps, $s \in S$;
- (iii) a *semitopological semigroup* if the map μ is separately continuous;
- (iv) a *topological semigroup* if the map μ is jointly continuous.

We usually omit the reference to μ and write simply S instead of (S, μ) . It goes without saying that every topological semigroup is also semitopological and every semitopological semigroup is both a right and left topological semigroup.

A topology τ on a semigroup S is called:

- a *semigroup topology* if (S, τ) is a topological semigroup;
- a *shift-continuous topology* if (S, τ) is a semitopological semigroup;
- an *left-continuous topology* if (S, τ) is a left topological semigroup;
- an *right-continuous topology* if (S, τ) is a right topological semigroup.

The bicyclic monoid $\mathcal{C}(p, q)$ is the semigroup with the identity 1 generated by two elements p and q subjected only to the condition $pq = 1$. The semigroup operation on $\mathcal{C}(p, q)$ is determined as follows:

$$q^k p^l \cdot q^m p^n = \begin{cases} q^{k-l+m} p^n, & \text{if } l < m; \\ q^k p^n, & \text{if } l = m; \\ q^k p^{l-m+n}, & \text{if } l > m. \end{cases}$$

We define the following subsets of the bicyclic monoid

$$\mathcal{C}_+(p, q) = \{q^i p^j \in \mathcal{C}(p, q) : i \leq j\} \quad \text{and} \quad \mathcal{C}_-(p, q) = \{q^i p^j \in \mathcal{C}(p, q) : i \geq j\}.$$

Proposition 3. $\mathcal{C}_+(p, q)$ and $\mathcal{C}_-(p, q)$ are anti-isomorphic submonoids of $\mathcal{C}(p, q)$.

Proposition 4. Green's relations $\mathcal{R}, \mathcal{L}, \mathcal{J}, \mathcal{D}$ and \mathcal{H} on monoids $\mathcal{C}_+(p, q)$ and $\mathcal{C}_-(p, q)$ coincide with the equality relation.

Theorem 5. Every Hausdorff left-continuous topology on the monoid $\mathcal{C}_+(p, q)$ is discrete.

Theorem 6. Every Hausdorff right-continuous topology on the monoid $\mathcal{C}_-(p, q)$ is discrete.

Example 7. There exists a non-discrete locally compact semigroup T_1 -topology τ on the monoid $\mathcal{C}_+(p, q)$.

Example 8. There exists a non-discrete compact shift-continuous T_1 -topology τ on the monoid $\mathcal{C}_+(p, q)$.

Proposition 9. If the monoid $\mathcal{C}_+(p, q)$ is a dense subsemigroup of a Hausdorff semitopological monoid S and $I = S \setminus \mathcal{C}_+(p, q) \neq \emptyset$ then I is a closed two-sided ideal of the semigroup S .

Example 10. There exists a compact Hausdorff topological monoid S which contains the monoid $\mathcal{C}_+(p, q)$ as a dense submonoid.

Also, we discuss under which conditions a shift-continuous T_1 -topology τ on the monoid $\mathcal{C}_+(p, q)$ is discrete.

REFERENCES

- [1] J. H. Carruth, J. A. Hildebrandt, and R. J. Koch, *The theory of topological semigroups*, Vol. I, Marcel Dekker, Inc., New York and Basel, 1983.

- [2] A. H. Clifford and G. B. Preston, *The algebraic theory of semigroups*, Vol. I, Amer. Math. Soc. Surveys **7**, Providence, R.I., 1961.
- [3] R. Engelking, *General topology*, 2nd ed., Heldermann, Berlin, 1989.
- [4] W. Ruppert, *Compact semitopological semigroups: an intrinsic theory*, Lect. Notes Math., **1079**, Springer, Berlin, 1984.

Existence and non-existence of cohomogeneity one Einstein metrics

Hanci Chi

(Xi'an Jiaotong-Liverpool University)

E-mail: hanci.chi@xjtlu.edu.cn

A Riemannian metric g is Einstein if $\text{Ric}(g) = \Lambda g$ for some constant Λ . A general existence theorem for homogeneous Einstein metrics was established in [WZ86]. It is natural to turn to the cohomogeneity one Einstein metrics, meaning that the principal orbit G/K is of codimension one. The cohomogeneity one condition reduces the Einstein equation to a system of ODEs. Previously known examples include [Pag78], [BB82], [KS86], [KS88], and [WW98]. Recently, we proved the existence of an Einstein metric on $\mathbb{H}\mathbb{P}^{m+1} \# \overline{\mathbb{H}\mathbb{P}^{m+1}}$ [Chi24], generalizing the result in [Böh98] to all higher dimensions.

We realize that the analytic techniques can be carried over to many other cohomogeneity one spaces. We develop two criteria to check the existence or non-existence of a cohomogeneity one Einstein metrics with a certain fixed principal orbit type. In particular, the principal orbit G/K is the total space of a sphere bundle over a singular orbit G/H , and both the fiber and the base space are irreducible. Each such a principal orbit is associated to a structural triple (d_1, d_2, A) , where $d_1 = \dim(H/K)$, $d_2 = \dim(G/H)$ and $A > 0$ is a constant obtained from the O'neil tensor in the theory of Riemannian submersion. The corresponding cohomogeneity one space, denoted as M , is a double disk bundle, where G/K collapses to G/H on two ends. The Einstein metric is obtained from the ansatz

$$dt^2 + f_1^2(t) b|_{\mathfrak{h}/\mathfrak{k}} + f_2^2(t) b|_{\mathfrak{g}/\mathfrak{h}}, \quad (1)$$

where t parametrizes the 1-dimensional orbit space and b is a background metric.

Our existence theorem is the following.

Theorem 1. *For any (d_1, d_2) with $d_2 \geq d_1 \geq 2$, there exists a constant $\chi_{d_1, d_2} \in \left(0, \frac{d_2(d_2-1)^2}{d_1^2(d_1 d_2 - d_2 + 4)}\right]$ such that if G/K is a principal orbit with $A \in [0, \chi_{d_1, d_2})$, then there is at least one cohomogeneity one Einstein metrics on M .*

The constant χ_{d_1, d_2} is an algebraic function in (d_1, d_2) , whose formula is very complicated in general. Nevertheless, we obtain many new examples of inhomogeneous Einstein metrics from previous works on homogeneous Einstein metrics including [DZ79], [WZ85], [Wan92], [DK08], [Nik16], [PZ21], and [LW24].

On the other hand, we also have the following non-existence theorem.

Theorem 2. *Define*

$$\Psi_{d_1, d_2} := \frac{(4(d_1 - 1)n^2 + d_2^2)(3n + d_1) d_2(d_2 - 1)^2}{(2n^2 + n + d_1)^2 d_1^2} \cdot \frac{1}{4(d_1 - 1)}.$$

If G/K is a principal orbit with $(d_1, d_2) \notin \{(2, 2), (2, 3), (2, 4)\}$ and $A \geq \Psi_{d_1, d_2}$, then there does not exist any G -invariant cohomogeneity one Einstein metrics on M from ansatz (1).

V. Dryuma <i>6D-Riemannian metric associated at the Navier-Stokes equations and its applications</i>	29
K. Eftekharinasab <i>Interplay of Global Implicit Functions and Critical Point Theory in Infinite Dimensional Spaces</i>	32
M. Egwe <i>Spherical Analysis on Fuzzy Lie Groups</i>	32
V. Fedorchuk, V. Fedorchuk <i>On the construction and classification of the common invariant solutions for the $(1 + 3)$-dimensional Euler-Lagrange-Born-Infeld and homogeneous Monge-Ampere equations</i>	33
B. Feshchenko <i>Normal forms of functions with degenerate critical points on surfaces whose stabilizers are homotopically non-trivial</i>	35
M. R. Formica <i>Nonlinear interpolation of α-Holderian mappings with applications to quasilinear PDEs</i>	36
J.F. Gálvez-Rodríguez, C. Martín-Aguado, M.A. Sánchez-Granero <i>Riemann Integration on a space with a fractal structure</i>	37
I. Gkeneralis <i>Topological rigidity of quoric manifolds</i>	39
O. Gok <i>On the multiplicative order convergence on Banach lattice f-algebras</i>	41
M. Golasiński <i>Octonionic Stiefel manifolds and vector bundles</i>	43
T. Goy <i>Some series involving central binomial coefficients</i>	44
O. Gutik <i>On non-topologizable semigroups</i>	46
H. Chi <i>Existence and non-existence of cohomogeneity one Einstein metrics</i>	48
I. Havrylenko, E. Petrov <i>Stability of vertical minimal surfaces in three-dimensional sub-Riemannian manifolds</i>	50
O. Ye. Hentosh <i>Rational factorization of Lax type flows in the space dual to the centrally extended Lie algebra of matrix super-integro-differential operators</i>	52
O. Hukalov, V. Gordevskyy <i>The some solution of the Bryan-Pidduck equation</i>	54
S. Ivković <i>Disjoint dynamical properties of wedge operators</i>	55
R. Ilemobade, T.G. Jaiyeola <i>On semi-symmetric (α, β, γ)-inverse quasigroup</i>	55
O. Karlova <i>About Rolewicz theorem on inversion of continuous bijection between F-spaces</i>	57
L. Fardigola, K. Khalina <i>On boundary controllability problems for the heat equation with variable coefficients on a half-axis</i>	58
N. Kitazawa <i>On explicit reconstruction of real algebraic maps locally like moment maps</i>	60
S.D. Koval, R.O. Popovych <i>Group classification of Kolmogorov backward equations with power diffusivity</i>	63