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- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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Morita equivalence of non-commutative Noetherian schemes

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This is a joint work with Igor Burban, see [1].

The classical Morita theorem (see, for instance, [3, Ch. 18]) claims that the categories of modules over rings A and B are equivalent if and only if there is a finitely generated projective generator P of the category of right A -modules such that $\text{End}_A P \simeq B$. Then this equivalence is established by the functor $P \otimes_A -$. If A and B are Noetherian, the same is the criterion of equivalence of their categories of finitely generated modules. On the other hand, Gabriel [2] proved that two Noetherian schemes X and Y are isomorphic if and only if the categories of coherent (or, which is the same, of quasi-coherent) sheaves of \mathcal{O}_X - and \mathcal{O}_Y -modules are equivalent. We present here a result which is, in some sense, a combination and generalization of these two classical theorems.

Definition 1. (1) A *non-commutative Noetherian scheme (NCNS)* is a pair $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$, where X is a separated Noetherian scheme and $\mathcal{O}_{\mathbb{X}}$ is a sheaf of \mathcal{O}_X -algebras which is coherent as a sheaf of \mathcal{O}_X -modules. We denote by $\text{Coh } \mathbb{X}$ and $\text{QCoh } \mathbb{X}$ respectively the categories of coherent and quasi-coherent sheaves of left $\mathcal{O}_{\mathbb{X}}$ -modules.

Note that the category $\text{QCoh } \mathbb{X}$ is locally Noetherian and $\text{Coh } \mathbb{X}$ is its subcategory of Noetherian objects. Therefore, they uniquely define each other.

- (2) Two NCNS \mathbb{X} and \mathbb{Y} are called *Morita equivalent* if the categories $\text{Coh } \mathbb{X}$ and $\text{Coh } \mathbb{Y}$ (or, which is the same, $\text{QCoh } \mathbb{X}$ and $\text{QCoh } \mathbb{Y}$) are equivalent.
- (3) A NCNS \mathbb{X} is called *central* if \mathcal{O}_X coincides with the center of $\mathcal{O}_{\mathbb{X}}$, i.e. for every point $x \in X$ the ring $\mathcal{O}_{X,x}$ is the center of the algebra $\mathcal{O}_{\mathbb{X},x}$.

Proposition 2. *For every NCNS $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ there is a Noetherian scheme Z and a morphism $\phi : Z \rightarrow X$ such that the NCNS $\tilde{\mathbb{X}} = (Z, \phi^* \mathcal{O}_{\mathbb{X}})$ is central and Morita equivalent to \mathbb{X} . Moreover, the ring of global sections $\Gamma(Z, \mathcal{O}_Z)$ is isomorphic to the center of the category $\text{Coh } \mathbb{X}$, i.e. the endomorphism ring of the identity functor $\text{id}_{\text{Coh } \mathbb{X}}$. If the scheme X is excellent, the morphism ϕ is finite.*

Thus, studying Morita equivalence, we can only consider central schemes. The following result is an analogue of the Gabriel's theorem.

Theorem 3. *If a NCNS $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ is central, the scheme X is determined by the category $\text{QCoh } \mathbb{X}$ (or, which is the same, by $\text{Coh } \mathbb{X}$) up to an isomorphism.*

Actually, we give an explicit construction that restores X from $\text{QCoh } \mathbb{X}$, namely, from the so called *spectrum* of this category in the sense of Gabriel [2], i.e. isomorphism classes of indecomposable injective objects. It is important that this construction also recovers affine open coverings of X .

Definition 4. A coherent sheaf of right $\mathcal{O}_{\mathbb{X}}$ -modules \mathcal{P} is called a *local progenerator* for \mathbb{X} if for every point $x \in X$ its stalk \mathcal{P}_x is a projective generator of the category of right $\mathcal{O}_{\mathbb{X},x}$ -modules.

Our main result is the following.

Theorem 5. *Let $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ and $\mathbb{Y} = (Y, \mathcal{O}_{\mathbb{Y}})$ be central NCNS. They are Morita equivalent if and only if there is an isomorphism $\phi : Y \rightarrow X$ and a local progenerator \mathcal{P} for \mathbb{X} such that $\phi^*(\mathcal{E}nd_{\mathcal{O}_{\mathbb{X}}} \mathcal{P}) \simeq \mathcal{O}_{\mathbb{Y}}$. Then this equivalence is established by the functor $\phi^*(\mathcal{P} \otimes_{\mathcal{O}_{\mathbb{X}}} -)$.*

Note that even if $X = Y$, the isomorphism ϕ need not be identity. If it is so, this equivalence is called *central*.

We also specialize this theorem for the case of *non-commutative curves*, where it gives a sort of “globalization” of the known results on the local–global correspondence from the theory of lattices over orders (or integral representations of rings).

Definition 6. A *non-commutative curve* is a NCNS $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ such that X is excellent and of pure dimension 1 and $\mathcal{O}_{\mathbb{X}}$ is *reduced*, i.e. contains no nilpotent ideals.

We always suppose \mathbb{X} *central* and *connected* (in the central case, it just means that X is connected). We denote by \mathcal{Q}_X the sheaf of fractions of \mathcal{O}_X and set $\mathcal{Q}_{\mathbb{X}} = \mathcal{Q}_X \otimes_{\mathcal{O}_X} \mathcal{O}_{\mathbb{X}}$. We denote $Q(X) = \Gamma(X, \mathcal{Q}_X)$ and $Q(\mathbb{X}) = \Gamma(X, \mathcal{Q}_{\mathbb{X}})$. Note that $Q(\mathbb{X})$ is a semisimple $Q(X)$ -algebra and for every closed point $x \in X$ the ring $\mathcal{O}_{\mathbb{X},x}$ is an $\mathcal{O}_{X,x}$ -order in this algebra. Since X is excellent, the set $\text{Sing}(\mathbb{X})$ of such closed points $x \in X$ that this order is not maximal is finite (it follows from [4, Ch. 6]).

Theorem 7. Let $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ and $\mathbb{Y} = (X, \mathcal{O}_{\mathbb{Y}})$ be two central non-commutative curves with the same central curve X . They are centrally Morita equivalent if and only if the following conditions are satisfied:

- the semisimple $Q(X)$ -algebras $Q(\mathbb{X})$ and $Q(\mathbb{Y})$ are centrally Morita equivalent;
- $\text{Sing}(\mathbb{X}) = \text{Sing}(\mathbb{Y})$;
- for every $x \in \text{Sing}(\mathbb{X})$ the $\mathcal{O}_{X,x}$ -orders $\mathcal{O}_{\mathbb{X},x}$ and $\mathcal{O}_{\mathbb{Y},x}$ (or, which is the same, their \mathfrak{m}_x -completions) are centrally Morita equivalent.

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