International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Definition 2. A *realization* of a Lie algebra $\mathfrak{g}$ in vector fields on M is a homomorphism

$$R\colon \mathfrak{g} \to \mathrm{Vect}(M).$$

Let us consider the algebra of endomorphisms $gl(V)$ of the vector space V and define a representation, which is closely related to Lie algebra module.

Definition 3. A *representation* of a Lie algebra $\mathfrak{g}$ is a homomorphism

$$\varphi\colon \mathfrak{g} \to gl(V).$$

Let us outline the algorithm in the case of realizations:

(1) Construct parameterized structure constants using the continuous function U, that do realize the desired contraction $C^{k'}_{\varepsilon,i'j'} := (U_\varepsilon)^i_{i'}(U_\varepsilon)^j_{j'}(U_\varepsilon{}^{-1})^{k'}_k C^k_{ij}$, where C^k_{ij} are structure constants of the initial Lie algebra.
(2) Calculate ε-dependent adjoint actions (using the structure constants $C^{k'}_{\varepsilon,i'j'}$), exponents and differential 1-forms: $\mathrm{ad}^\varepsilon e_i$, $\exp(-x_i \mathrm{ad}^\varepsilon e_i)$, $\omega^\varepsilon(x)$.
(3) Find the inverse transformation to obtain the vector fields $\xi^\varepsilon(x) = (\omega^\varepsilon(x))^{-1}$, that are the parameterized realization that do contracts to the realization of the contracted Lie algebra.

To conclude let us mention that contraction of the fixed realization or representation of a Lie algebra is more complicated task. Namely, in the case of realization, we first have to define it's subalgebra (studying the kernel of the linear operator in the initial point), then we have to find the equivalence transformations to the canonical realization. After that we can apply our algorithm and complete it by the inverse of the equivalence transformations.

References

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Geodesic orbit pseudo Riemannian nilmanifolds

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We know that in the Riemannian case, (i) for every homogeneous space, there is a reductive decomposition at the level of Lie algebras, (ii) the isometry group of a simply connected nilmanifold is the semidirect product of isometric automorphisms and translations (Wolf/Wilson), and (iii) geodesic orbit nilmanifolds are necessarily two-step nilpotent or abelian (Gordon). Neither of this is true in pseudo-Riemannian signature. However, it turns out that in low signature, some results may still be "rescued". This is a joint work (which is partially still in progress) with Joe Wolf, Zhiqi Chen and Shaoxiang Zhang.