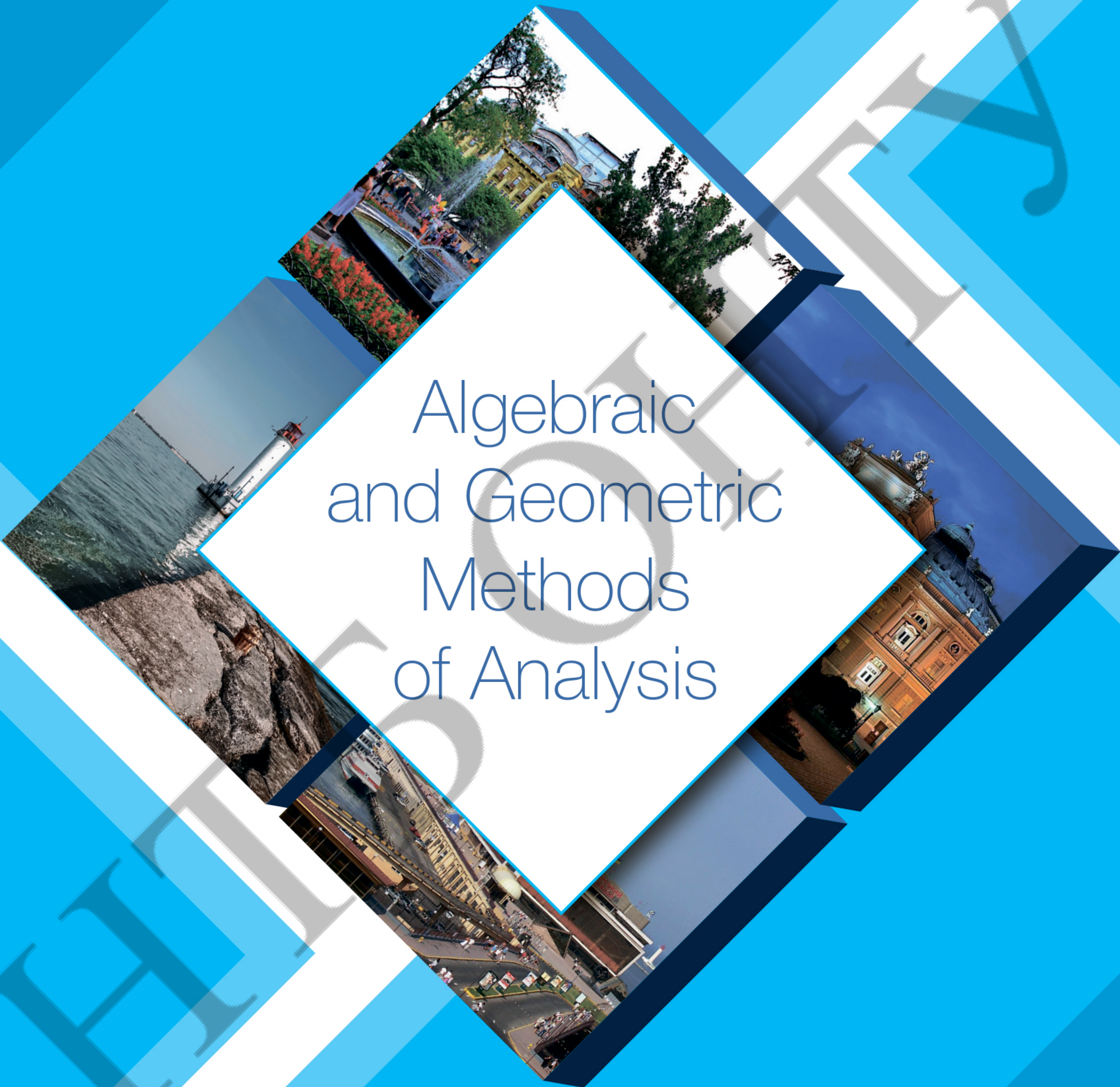
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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If we consider a conformal mapping $f : (V^{1,3}, g) \to (\tilde{V}^{1,3}, \tilde{g})$, i.e. $\tilde{g}_{ij} = e^{2\varphi(x)} g_{ij}$, then the vielbein is transformed as

$$\tilde{t}_i^a(x) = e^{\varphi(x)} t_i^a(x). \tag{2}$$

Here $\varphi(x)$ is some function. Under a conformal transformation the spin connection transforms as

$$\tilde{\omega}_{kab} = \omega_{kab} + t_{ka}\varphi_b - t_{kb}\varphi_a.$$

Here $\varphi_b = \partial_b \varphi = t_b^j \partial_j \varphi$. Hence for the spin-affine connection Γ_k we get:

$$\tilde{\Gamma}_k = \Gamma_k - \frac{1}{4}(t_{ka}\varphi_b - t_{kb}\varphi_a)\gamma^{ab} = \Gamma_k - \frac{1}{2} t_{ka}\varphi_b \gamma^{ab}. \tag{3}$$

On the other hand, the stress-energy tensor for the spinor field ($s = \frac{1}{2}$) in a spacetime $(V^{1,3}, g)$ we could calculate by the formula [3]:

$$T_{jk} = \frac{i}{2}\big(\overline{\psi}\gamma_{(j}\nabla_{k)}\psi - (\nabla_{(j}\overline{\psi})\gamma_{k)}\psi\big), \tag{4}$$

where $\gamma_j = \gamma_a t_j^a(x)$. Taking into account (2), (3), (4) we obtain the transformed stress-energy tensor:

$$\tilde{T}_{jk} = e^{\varphi(x)}\big(T_{jk} - \frac{i}{4}\big(\overline{\psi}\gamma_j t_{ka}\varphi_b\gamma^{ab}\psi + \overline{\psi}\gamma_k t_{ja}\varphi_b\gamma^{ab} + \overline{\psi} t_{ka}\varphi_b\gamma^{ab}\gamma_j + \overline{\psi} t_{ja}\varphi_b\gamma^{ab}\gamma_k\psi\big)\big),$$

However we have the scalar which is preserved under conformal mappings:

$$|A|^2 = A^i g_{ij} A^j = \overline{\psi}\gamma^i\psi g_{ij}\overline{\psi}\gamma^j\psi,$$

where $A^i = \overline{\psi}\gamma^i\psi$ is so called four-dimensional current of the spinor field ψ.

References

[1] P. Collas, D. Klein, *The Dirac Equation in Curved Spacetime*, SpringerBriefs in Physics, 2019.
[2] T. Yoshiaki, *Introduction to Supergravity*, Springer Tokyo Heidelberg New York Dordrecht London, 2018.
[3] N. D. Birrell, P. C. W. Davies,*Quantum Fields in Curved Space*, Cambridge University Press, England, 1982.

On topologization of the bicyclic monoid

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In this paper we shall follow the terminology of [2, 4, 5, 6].

A semigroup S is called *inverse* if for any element $x \in S$ there exists a unique $x^{-1} \in S$ such that $xx^{-1}x = x$ and $x^{-1}xx^{-1} = x^{-1}$. The element x^{-1} is called the *inverse of* $x \in S$. If S is an inverse semigroup, then the function inv: $S \to S$ which assigns to every element x of S its inverse element x^{-1} is called the *inversion*. On an inverse semigroup S the semigroup operation determines the *natural partial order* $\preccurlyeq$ on S: $s \preccurlyeq t$ if and only if there exists $e \in E(S)$ such that $s = te$.

A topology τ on a semigroup S is called:

- a *semigroup* (*shift-continuous*) topology if (S, τ) is a topological (semitopological) semigroup;
- an *inverse semigroup* topology if (S, τ) is a topological inverse semigroup;

- an *inverse shift-continuous* topology if (S,τ) is a semitopological semigroup with continuous inversion.

The bicyclic monoid $\mathcal{C}(p,q)$ is the semigroup with the identity 1 generated by two elements p and q subjected only to the condition $pq=1$. The bicyclic monoid admits only the discrete semigroup Hausdorff topology [3]. Bertman and West in [1] extended this result for the case of Hausdorff semitopological semigroups.

We construct two non-discrete inverse semigroup T_1-topologies and a compact inverse shift-continuous T_1-topology on the bicyclic monoid $\mathcal{C}(p,q)$. Also we give conditions on a T_1-topology τ on $\mathcal{C}(p,q)$ to be discrete.

Theorem 1. *Every shift-continuous Baire T_1-topology τ on the bicyclic monoid $\mathcal{C}(p,q)$ is discrete.*

Theorem 2. *Let τ be an inverse semigroup T_1-topology on $\mathcal{C}(p,q)$. If there exists a point $q^ip^j \in \mathcal{C}(p,q)$ such that the space $\updownarrow_{\preccurlyeq} q^ip^j$ is quasi-regular at q^ip^j, then τ is discrete.*

Theorem 3. *Let τ be a shift-continuous T_1-topology on the bicyclic monoid $\mathcal{C}(p,q)$ such that the maps $\mathcal{C}(p,q) \to E(\mathcal{C}(p,q))$, $x \mapsto xx^{-1}$ and $\mathcal{C}(p,q) \to E(\mathcal{C}(p,q))$, $x \mapsto x^{-1}x$ are continuous. If there exists a point $q^ip^j \in \mathcal{C}(p,q)$ such that the space $\updownarrow_{\preccurlyeq} q^ip^j$ is semiregular at q^ip^j, then τ is discrete.*

References

[1] M. O. Bertman and T. T. West, *Conditionally compact bicyclic semitopological semigroups*, Proc. Roy. Irish Acad. **A76** (1976), no. 21–23, 219–226.

[2] J. H. Carruth, J. A. Hildebrant, and R. J. Koch, *The theory of topological semigroups*, Vol. I, Marcel Dekker, Inc., New York and Basel, 1983.

[3] C. Eberhart and J. Selden, *On the closure of the bicyclic semigroup*, Trans. Amer. Math. Soc. **144** (1969), 115–126.

[4] R. Engelking, *General topology*, 2nd ed., Heldermann, Berlin, 1989.

[5] M. Lawson, *Inverse semigroups. The theory of partial symmetries*, Singapore: World Scientific, 1998.

[6] W. Ruppert, *Compact semitopological semigroups: an intrinsic theory*, Lect. Notes Math., **1079**, Springer, Berlin, 1984.

An Application to Sasaki Extremal metrics via the Berglund-Hübsch rule

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Sasaki-extremal metrics were introduced in [1] as a generalization of metrics with constant scalar curvature, which is obstructed by the Futaki invariant. On this talk we exhibit examples of homotopy spheres and rational homology spheres realized as links of chain-cycle polynomials that do not admit Sasaki extremal metrics in the whole Sasaki cone, which has dimension greater than one. For this, we consider links that are given as 2-fold branched covers of S^9 whose branching loci are rational homology 7-spheres which are links of certain invertible polynomials of chain-cycle type studied in [5] and later in [4] through the Berglund-Hübsch rule of classical mirror symmetry. In [2], Boyer and van Coevering defined a relative version of the K-stability of Collins and Székelyhidy [6, 7] and obtain the first examples of Sasaki manifolds with Sasaki cone of dimension greater

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