

International scientific conference
**«Algebraic and geometric
methods of analysis»**

Book of abstracts



May 30 - June 4, 2018,
Odesa,
Ukraine

<https://www.imath.kiev.ua/~topology/conf/agma2018>

Foliations with leaves of non-positive curvature and bounded total curvature on closed 3-manifolds

Dmitry V. Bolotov

(B. Verkin ILTPE of NASU, 47 Nauky Ave., Kharkiv, 61103, Ukraine)

E-mail: bolotov@ilt.kharkov.ua

Let (M, g) be a complete non-compact surface equipped with a smooth riemannian metric. The total curvature of M is the improper integral $\int_M K d\mu$ of the Gaussian curvature K with respect to the volume element $d\mu$ of (M, g) . It is said that M admits total curvature if for any compact exhaustion Ω_i of M , the limit

$$\lim_{i \rightarrow +\infty} \int_{\Omega_i} K d\mu = \int_M K d\mu, \quad (1)$$

exists. In [1] Cohn-Vossen proved that $\int_M K d\mu \leq 2\pi\chi(M)$, where $\chi(M)$ is the Euler characteristic of M . Huber in [2] states that if

$$\int_M K_- < \infty, \quad (2)$$

where $K_- = \max\{-K, 0\}$, then $\int_M K d\mu$ exists and M is homeomorphic to a compact Riemann surface with finitely many punctures, i.e. M has a finite topology. Hartman in [3] under the assumption (2) proved that the area of a geodesic ball of radius r at a fixed point must grow at most quadratically in r . Note also that Li proved in [4] that if M has at most quadratic area growth, finite topology and the Gaussian curvature of M is either non-positive or non-negative near infinity of each end, then M must have finite total curvature.

The following theorem describes a topological structure of riemannian 3-Manifolds admitting codimension one C^2 -foliations \mathcal{F} with leaves which have both non-positive curvature and bounded total curvature in the induced riemannian metric.

Theorem 1. *Let \mathcal{F} be a transversaly orientable C^2 -foliation of a closed orientable riemannian 3-Manifold M . Suppose, that the leaves of \mathcal{F} have non-positive curvature and admit a finite total curvature in the induced riemannian metric. Then the following holds:*

- (1) M is aspherical;
- (2) \mathcal{F} is a foliation almost without holonomy;
- (3) At least one of the following holds:
 - (a) \mathcal{F} is a surface bundle over the circle with the fiber genus $g \geq 1$;
 - (b) M is divided by a finite set of compact surfaces $\{K_i\}$, which are homeomorphic to torus T^2 , into pieces $\{A_j\}$, which are fibered over the circle. This division defines a graph G of fundamental groups $\pi_1(A_j)$ and $\pi_1(K_i)$, where vertexes of G correspond to the $\{A_j\}$ and edges of G correspond to the tori $\{K_i\}$ and the fundamental group $\pi_1(M)$ is isomorphic to a fundamental group of the graph G ;
- (4) \mathcal{F} is a flat foliation (i.e. all leaves of \mathcal{F} are flat) iff M is either torical bundle or torical semi-bundle.

Conversely, let M be such as described in (3) above. Then M admits a riemannian metric and transversaly orientable foliation with leaves of non-positive curvature and finite total curvature in the induced metric.

REFERENCES

- [1] S. Cohn-Vossen *Kürzeste Wege und Totalkrümmung auf Flächen*, Compositio Math. **2** (1935), 69–133.
- [2] A. Huber, *On subharmonic functions and differential geometry in the large*, Comment. Math. Helv. **32** (1957), 13–72.

- [3] Ph. Hartman, *Geodesic parallel coordinates in the large*, Amer. J. Math. **86** (1964), 705–727.
- [4] P. Li, *Complete surfaces of at most quadratic area growth*, Comment. Math. Helv. **72(1)** (1997), 67–71.

Зміст

N. Aygor, H. Burhanzade <i>Secondary school students' misconceptions about linear algebra</i>	3
S. Bardyla, H. Kvasnytsia <i>Semitopological graph inverse semigroups</i>	4
B. A. Bhayo <i>On inequalities of generalized elliptic integrals</i>	5
Bodzioch M., Choiński M., Foryś U. <i>A criss-cross model of tuberculosis for heterogenous population</i>	6
Bolotov D. V. <i>Foliations with leaves of non-positive curvature and bounded total curvature on closed 3-manifolds</i>	7
E. Bonacci <i>Algebraic and geometric questions about a 6D physics</i>	9
F. Bulnes <i>Mukai-Fourier Transform in Derived Categories to Solutions of the Field Equations: Gravitational Waves as Oscillations in the Space-Time Curvature/Spin IV</i>	10
H. Burhanzade, N. Aygor <i>A study on the teaching methods in determinants</i>	12
Damla Yaman <i>Order continuity properties of lattice ordered algebras</i>	13
Denega I. <i>Problem on non-overlapping polycylindrical domains with poles on the boundary of a polydisk</i>	14
A. Dudko, V. Pivovarchik <i>Inverse three spectra problem for a Stieltjes string with the Neumann boundary conditions</i>	16
Eftekharinasab K. <i>On the existence of a global diffeomorphism between Fréchet spaces</i>	18
Glazunov N. <i>Class groups of rings with divisor theory, L-functions and moduli spaces</i>	19
O. Gok <i>b-bimorphisms</i>	21
Gül E. <i>On the second regularized trace formula for a differential operator with unbounded coefficients</i>	22
Hentosh O. Ye., Prykaratsky Ya. A. <i>The Lie-algebraic structure of the Lax-Sato integrable superanalogs for the Liouville heavenly type equations</i>	24
V. Herasymov <i>In a natural topological sense a typical linear nonhomogeneous differential equation in the ring $Z[[x]]$ has no solutions from $Z[[x]]$.</i>	26
Juraev D. A. <i>On the Cauchy problem for matrix factorizations of the Helmholtz equation</i>	27
M. E. Kansu <i>Macroscopic electromagnetism via complex quaternions</i>	29
Vladimir V. Kisil <i>An extension of Möbius–Lie geometry with conformal ensembles of cycles</i>	30
Konovenko N., Lychagin V. <i>Rational differential invariants for oriented primary visual cortex</i>	32