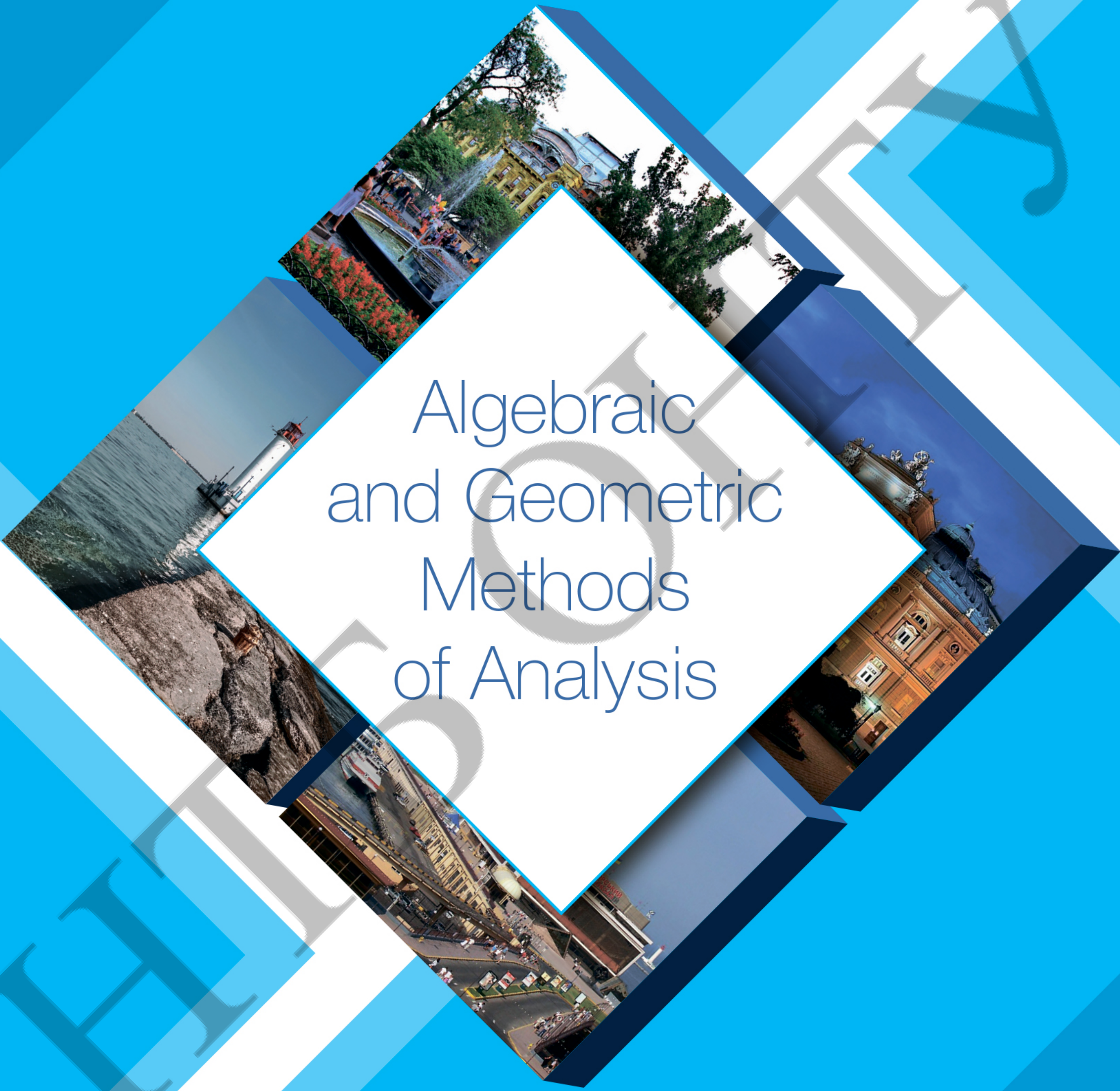
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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on nonlinear interpolation of α-Hölderian mappings $\mathcal{T}$ by studying the action of the mappings $\mathcal{T}$ on K-functionals and between interpolation spaces with logarithm functors. Therefore, we identify some interpolation spaces using couples of Lebesgue or Lorentz spaces, recovering spaces as Lorentz–Zygmund spaces or $G\Gamma$-gamma spaces.

We apply these results to obtain regularity on the gradient of the weak or entropic-renormalized solution u to quasilinear equations of the form

$$-\mathrm{div}(\widehat{a}(\nabla u)) + V(x;u) = f, \qquad u = 0 \text{ on } \partial\Omega, \tag{1}$$

associated to the Dirichlet homogeneous condition on the boundary, where Ω is a bounded smooth domain of $\mathbb{R}^n$, $\widehat{a}(\nabla u) = |\nabla u|^{p-2}\nabla u$, V is a nonlinear potential and f belongs to non-standard spaces like Lorentz-Zygmund spaces. We also show that the mapping $\mathcal{T}: \mathcal{T}f = \nabla u$ is locally or globally α-Hölderian under suitable values of α and appropriate assumptions on V and $\widehat{a}$.

Furthermore, also the anisotropic version or the variable exponents version of the Laplacian are considered.

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Riemann Integration on a space with a fractal structure

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In this work we start developing a Riemann-type integration theory on spaces which are equipped with a fractal structure (see [1] for more details). The definition of a fractal structure is the next one:

Definition 1. A fractal structure $\mathbf{\Gamma}$ on a set X is a countable family of coverings $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ such that Γ_{n+1} is a strong refinement of Γ_n for each $n \in \mathbb{N}$. Γ_2 is said to be a strong refinement of Γ_1 if Γ_2 is a refinement of Γ_1 (that is, each element of Γ_2 is contained in some element of Γ_1) and for each $B \in \Gamma_1$ it holds that $B = \bigcup\{A \in \Gamma_2 : A \subseteq B\}$. Cover Γ_n is called level n of the fractal structure.

We require to define a concept first:

Definition 2. Let $(X, \mathcal{S}, \mu)$ be a measure space and $\mathbf{\Gamma}$ be a fractal structure on X. $\mathbf{\Gamma}$ is said to be μ-disjoint if the following conditions hold:

(1) $\Gamma_n \subseteq \mathcal{S}$ is countable for each $n \in \mathbb{N}$.
(2) $\mu(B \cap J) = 0$ for each $B, J \in \Gamma_n$ such that $B \neq J$ and each $n \in \mathbb{N}$.
(3) $\mu(A) < \infty$ for each $A \in \Gamma_n$ and each $n \in \mathbb{N}$.

Next, we define the Darboux sums with respect to a measure and a fractal structure:

Definition 3. Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ-disjoint fractal structure, and $f : X \to \mathbb{R}$ be a bounded function. Then, for each $J \in \Gamma_n$, we set $m(f; J) = \inf\{f(x) : x \in J\}$ and $M(f; J) = \sup\{f(x) : x \in J\}$, so that the lower and upper Darboux sums with respect to μ for each level of the fractal structure are given by

$$L(f; \Gamma_n, \mu) = \sum_{J \in \Gamma_n} m(f; J)\mu(J) \qquad \text{and} \qquad U(f; \Gamma_n, \mu) = \sum_{J \in \Gamma_n} M(f; J)\mu(J).$$

The lower and upper Riemann integrals with respect to a measure and a fractal structure are defined as follows:

Definition 4. Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ-disjoint fractal structure on X, and $f : X \to \mathbb{R}$ be a bounded function. We define the lower and upper Riemann integrals of f with respect to μ and $\mathbf{\Gamma}$ on X as follows:

(1) Upper Riemann integral of f with respect to μ and $\mathbf{\Gamma}$:

$$\overline{\int_X}^{(\mu,\mathbf{\Gamma})} f := \inf\{U(f; \Gamma_n; \mu) : n \in \mathbb{N}\} = \lim_n U(f; \Gamma_n; \mu).$$

(2) Lower Riemann integral of f with respect to μ and $\mathbf{\Gamma}$:

$$\underline{\int_X}^{(\mu,\mathbf{\Gamma})} f := \sup\{L(f; \Gamma_n; \mu) : n \in \mathbb{N}\} = \lim_n L(f; \Gamma_n; \mu).$$

Now we give the defition of a Riemann-integrable function.

Definition 5. Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ-disjoint fractal structure on X and $f : X \to \mathbb{R}$ be a bounded function. f is said to be Riemann-integrable with respect to μ and $\mathbf{\Gamma}$ on X if $\overline{\int_X}^{(\mu,\mathbf{\Gamma})} f$ is finite and $\underline{\int_X}^{(\mu,\mathbf{\Gamma})} f = \overline{\int_X}^{(\mu,\mathbf{\Gamma})} f$.

If f is Riemann-integrable with respect to μ and $\mathbf{\Gamma}$ on X, we define the Riemann integral of f with respect to μ and $\mathbf{\Gamma}$ on X, $\int_X^{(\mu,\mathbf{\Gamma})} f$, by $\int_X^{(\mu,\mathbf{\Gamma})} f = \underline{\int_X}^{(\mu,\mathbf{\Gamma})} f = \overline{\int_X}^{(\mu,\mathbf{\Gamma})} f$. We denote by $R(X; \mu; \mathbf{\Gamma})$ the set of Riemann-integrable functions with respect to μ and $\mathbf{\Gamma}$ on X.

The next step is defining the Riemann sum relative to a collection of points in a certain level of $\mathbf{\Gamma}$.

Definition 6. Let $\mathbf{\Gamma}$ be a fractal structure on a space X such that Γ_n is countable for each $n \in \mathbb{N}$. A selection for Γ_n is a collection of points $\xi := (x_A)_{A \in \Gamma_n}$ such that $x_A \in A$ for each $A \in \Gamma_n$.

Definition 7. Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ-disjoint fractal structure on X and $f : X \to \mathbb{R}$ be a bounded function. Let $n \in \mathbb{N}$ and $\xi = (x_A)_{A \in \Gamma_n}$ be a selection for Γ_n. The Riemann sum for f relative to Γ_n, ξ and μ is defined as $S(f; \Gamma_n; \xi; \mu) := \sum_{A \in \Gamma_n} f(x_A)\mu(A)$.

The following theorem is analogous to the Riemann's Theorem in $\mathbb{R}^n$, but for bounded functions defined on a space with a μ-disjoint fractal structure.

Theorem 8. *Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ be a μ-disjoint fractal structure on X, $f : X \to \mathbb{R}$ be a bounded function and $C \in \mathbb{R}$. The following statements are equivalent:*

(1) $f \in R(X; \mu; \mathbf{\Gamma})$ and $\int_X^{(\mu,\mathbf{\Gamma})} f = C$.
(2) Given $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $|C - S(f; \Gamma_n; \xi_n; \mu)| < \varepsilon$ for each $n \geq n_0$ and each selection for Γ_n, ξ_n.
(3) Given $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that $|C - S(f; \Gamma_n; \xi; \mu)| < \varepsilon$ for each selection for Γ_n, ξ.
(4) $S(f; \Gamma_m; \xi_m; \mu) \overset{m\to\infty}{\longrightarrow} C$ for each sequence (ξ_m) such that ξ_m is a selection for Γ_m for each $m \in \mathbb{N}$.

The next result is crucial in order to justify that the Riemann integral of a bounded function with respect to a measure and a fractal structure does not depend on the fractal structure.

Proposition 9. *Let $(X, \mathcal{S}, \mu)$ be a measure space, $\mathbf{\Gamma} = \{\Gamma_n : n \in \mathbb{N}\}$ and $\mathbf{\Gamma}^* = \{\Gamma_n^* : n \in \mathbb{N}\}$ be two μ-disjoint fractal structures on X and $f : X \to \mathbb{R}$ be a bounded function. If $f \in R(X; \mu; \mathbf{\Gamma})$ and $f \in R(X; \mu; \mathbf{\Gamma}^*)$, then $\int_X^{(\mu,\mathbf{\Gamma})} f = \int_X^{(\mu,\mathbf{\Gamma}^*)} f$.*

Hence, it does make sense to introduce the following concept:

Definition 10. Let $(X, \mathcal{S}, \mu)$ be a measure space and $f : X \to \mathbb{R}$ be a bounded function. f is said to be μ-Riemann-integrable if there exists a μ-disjoint fractal structure $\mathbf{\Gamma}$ on X such that f is Riemann-integrable on X with respect to μ and $\mathbf{\Gamma}$. Moreover, if so, the integral is defined as $\int_X^{\mu} f = \int_X^{(\mu,\mathbf{\Gamma})} f$.

Proposition 11. *Let $(X, \mathcal{S}, \mu)$ be a finite measure space and $f : X \to \mathbb{R}$ be a bounded measurable function. Then $f \in R(X; \mu)$ and $\int_X^{\mu} f = \int f d\mu$.*

Hence, if $\mathbf{\Gamma}$ is a μ-disjoint fractal structure on X such that f is Riemann-integrable with respect to μ and $\mathbf{\Gamma}$, we can calculate $\int f d\mu$ as $\int_X^{(\mu,\mathbf{\Gamma})} f$. It also follows that if μ is a finite measure on the Borel σ-algebra of a topological space X and $f : X \to \mathbb{R}$ is a bounded continuous map, then f is μ-Riemann-integrable on X.

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Topological rigidity of quoric manifolds

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The basic problem in Geometric Topology is the topological classification of manifolds, spaces that are locally like the usual Euclidean spaces, like the surfaces. More precisely, we study manifolds that have the same algebraic properties (homotopy equivalences) and we would like to show that they are equivalent (homeomorphic). There are a lot of conjectures towards this direction with the strongest being the Isomorphism Conjecture of Farrell-Jones. Furthermore, there are the