International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Bolotov D.** (*Kharkiv, Ukraine*)
- **Bondarenko V.** (*Kyiv, Ukraine*)
- **Boychuk O.** (*Kyiv, Ukraine*)
- **Boyko V.** (*Kyiv, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Dorogovtsev A.** (*Kyiv, Ukraine*)
- **Drozd Yu.** (*Kyiv, Ukraine*)
- **Gerasymenko V.** (*Kyiv, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Kiosak V.** (*Odesa, Ukraine*)
- **Kochubei A.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Mikhailets V.** (*Kyiv, Ukraine*)
- **Ostrovskyi V.** (*Kyiv, Ukraine*)
- **Petravchuk A.** (*Kyiv, Ukraine*)
- **Plaksa S.** (*Kyiv, Ukraine*)
- **Portenko M.** (*Kyiv, Ukraine*)
- **Pratsiovytyi M.** (*Kyiv, Ukraine*)
- **Savchenko O.** (*Kherson, Ukraine*)
- **Romanyuk A.** (*Kyiv, Ukraine*)
- **Timokha O.** (*Kyiv, Ukraine*)

Organizing commitee

- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Osadchuk Ye.** (*Odesa, Ukraine*)
- **Sergeeva O.** (*Odesa, Ukraine*)

The solution to this equation will be look for in the next form

$$f(t,x,V)=\sum_{i=1}^{\infty}\varphi_i(t,x)M_i(t,x,V). \tag{4}$$

where $M_i(t,x,V)$ are the exact solutions of the equation (1)-(3)

$$D(M_i)=Q(M_i,M_i)=0$$

and the coefficient functions $\varphi_i(t,x)$ are nonnegative smooth functions on $\mathbb{R}^4$ and $\varphi_i(t,x)\not\equiv 0$.

As a value of the deviation between the parts of equation (1) we will consider a uniform-integral error of the form

$$\Delta=\Delta(\beta_i)=\sup_{(t,x)\in\mathbb{R}^4}\int_{\mathbb{R}^3}\big|D(f)-Q(f,f)\big|dV. \tag{5}$$

In the paper [2], several cases of coefficient functions $\varphi_i(t,x)$ were obtained for which the deviation (5) can be done arbitrarily small. This is possible thanks to a special selection of hydrodynamic flow parameters.

References

[1] C. Cercignani. Theory and Application of the Boltzmann Equation. *Scottish Academic Press*, Edinburgh, 1975.

[2] O.O. Hukalov, V.D. Gordevskyy. The Interaction of an Infinite Number of Eddy Flows for the Hard Spheres Model. *Journal of Mathematical Physics, Analysis, Geometry*, 2: 163–174, 2021.

Semi-Fredholm theory in unital C^*-algebras

Stefan Ivković

(Mathematical Institute of the Serbian Academy of Sciences and Arts, Kneza Mihaila 36, Beograd, Serbia)

E-mail: `stefan.iv10@outlook.com`

The Fredholm and semi-Fredholm theory on Hilbert and Banach spaces started by studying the integral equations introduced in the pioneering work by Fredholm in 1903 in [5]. After that, the abstract theory of Fredholm and semi-Fredholm operators on Hilbert and Banach spaces was further developed in numerous papers and books such as [1], [2] and [14]. In addition to classical semi-Fredholm theory on Hilbert and Banach spaces, several generalizations of this theory have been considered. Breuer for example started the development of Fredholm theory in von-Neumann algebras as a generalization of the classical Fredholm theory for operators on Hilbert spaces. In [3] and [4] he introduced the notion of a Fredholm operator in a von Neumann algebra and established its main properties. On the other hand, Fredholm theory on Hilbert C^*-modules as another generalization of the classical Fredholm theory on Hilbert spaces was started by Mishchenko and Fomenko. In [13] they introduced the notion of a Fredholm operator on the standard Hilbert C^*-module and proved a generalization in this setting of some of the main results from the classical Fredholm theory. In [6], [7], [8], [9] and [10] we went further in this direction and defined semi-Fredholm and semi-Weyl operators on Hilbert C^*-modules. We investigated and proved several properties of these new semi- Fredholm operators on Hilbert C^*-modules as a generalization of the results from the classical semi-Fredholm theory on Hilbert and Banach spaces. The interest for considering these generalizations comes from the theory of pseudo differential operators acting on manifolds. The classical theory can be applied in the case of compact manifolds, but not in the case of non-compact ones. Even operators on Euclidian spaces are hard to study, for example Laplacian is not Fredholm. Kernels and cokernels of many operators are infinite dimensional Banach spaces, however, they may also at the same time be finitely generated Hilbert modules over some appropriate C^*-algebra. Similarly, orthogonal projections onto kernels and cokernels of many bounded linear operators on Hilbert spaces are not finite rank projections in the classical sense, but they are still finite projections in an appropriate von Neumann algebra. Therefore, many operators that are not semi-Fredholm in the classical sense may become semi-Fredholm in a more general sense if we consider them as operators on Hilbert C^*-modules or as elements of an appropriate von Neumann algebra. Hence, by studying these generalized semi-Fredholm operators, we get a proper extension of the classical semi-Fredholm theory to new classes of operators.

Now, Kečkić and Lazović in [12] established an axiomatic approach to Fredholm theory. They introduced the notion of a finite type element in a unital C^*-algebra which generalizes the notion of the compact operator on the standard Hilbert C^*-module and the notion of a finite operator in a properly infinite von Neumann algebra. They also introduced the notion of a Fredholm type element with respect to the ideal of these finite type elements. This notion is at a same time a generalization of the classical Fredholm operator on a Hilbert space, Fredholm C^*-operator on the standard Hilbert C^*-module defined by Mishchenko and Fomenko and the Fredholm operator on a properly infinite von Neumann algebra defined by Breuer. The index of this Fredholm type element takes values in the K-group. They showed that the set of Fredholm type elements in a unital C^*-algebra is open in the norm topology and they proved a generalization of the Atkinson theorem. Moreover, they proved the multiplicativity of the index in the K-group and that the index is invariant under perturbations of Fredholm type elements by finite type elements.

In this talk we will present the results from [11] regarding semi-Fredholm theory in unital C^*-algebras as a continuation of the approach by Kečkić and Lazović on Fredholm theory in unital C^*-algebras. We will introduce the notion of a semi-Fredholm type element and a semi-Weyl type element with respect to the ideal of finite type elements and obtain a generalization in this setting of several results from the classical semi-Fredholm and semi- Weyl theory of operators on Hilbert spaces. The motivation for this research is not only developing an abstract, axiomatic semi-Fredholm theory in unital C^*-algebras, but also deriving an extension of Breuer‘s Fredholm theory to semi-Fredholm and semi-Weyl theory in properly infinite von Neumann algebras by applying our results to this special case. In the first part of the talk we will present the results in abstract semi-Fredholm theory and semi-Weyl theory in unital C^*-algebras, whereas in the second part of the talk we will focus on the applications of these results to the concrete case of properly infinite von Neumann algebras.

References

[1] P. Aiena, *Fredholm and Local Spectral Theory, with Applications to Multipliers*, Kluwer (2004), ISBN 978-1-4020-2525-9

[2] P. Aiena, *Fredholm and Local spectral Theory II, Lecture Notes in Mathematics*, 2235, (2018), ISBN 978-3-030-02266-2

[3] M. Breuer, *Fredholm theories in von Neumann algebras. I*, Math. Ann. **178**, 243–254 (1968). https://doi.org/10.1007/BF01350663

[4] M. Breuer, *Fredholm theories in von Neumann algebras. II*, Math. Ann. **180**, 313–325 (1969). https://doi.org/10.1007/BF01351884

[5] E. I. Fredholm, *Sur une classe d'equations fontionnelles*, Acta Math. **27** (1903), 365–390.

[6] S. Ivković, *Semi-Fredholm theory on Hilbert C*-modules,* Banach J. Math. Anal., **13** (4) , 989-1016 October (2019) doi:10.1215/17358787-2019-0022. https://projecteuclid.org/euclid.bjma/1570608171

[7] S. Ivković, *On operators with closed range and semi-Fredholm operators over W*-algebras*, Russ. J. Math. Phys. **27**, 48-60 (2020) http://link.springer.com/article/10.1134/S1061920820010057

[8] S. Ivković, *On various generalizations of semi-A-Fredholm operators*, Complex Anal. Oper. Theory **14**, 41 (2020). https://doi.org/10.1007/s11785-020-00995-3

[9] S. Ivković, *On Upper Triangular Operator* 2×2 *Matrices Over C*-Algebras*, FILOMAT, (2020), vol. **34** no. 3, 691-706. https://doi.org/10.2298/FIL2003691I

[10] S. Ivković, *On Drazin invertible C*-operators and generalized C*-Weyl operators*, Ann. Funct. Anal. **14**, 36 (2023). https://doi.org/10.1007/s43034-023-00258-0

[11] S. Ivković, *Semi-Fredholm theory in C*-algebras*, https://arxiv.org/abs/2002.04905

[12] D. J. Kečkić, Z. Lazović, *Fredholm operators on C*-algebras.* ActaSci.Math. **83** , 629-655 (2017). https://doi.org/10.14232/actasm-015-526-5

[13] A. S. Mishchenko, A.T. Fomenko, *The index of eliptic operators over C*-algebras*, Izv. Akad. Nauk SSSR Ser. Mat. **43** (1979), 831–859; English transl., Math. USSR-Izv.**15** (1980) 87–112.

[14] S. C. Živković Zlatanović, *An Introduction into Fredholm Theory and Generalized Drazin-Riesz Invertible Operators*, **20** (28), pp. 113–198, Matematički institut SANU, Beograd (2022). ISSN: 0351-9406

On some non-associative hyper-algebraic structures

Temitope Gbolahan Jaiyeola
(Department of Mathematics, Obafemi Awolowo University, Ile Ife 220005, Nigeria)
E-mail: tjayeola@oauife.edu.ng

Kehinde Gabriel Ilori
(Department of Mathematics, Obafemi Awolowo University, Ile Ife 220005, Nigeria)
E-mail: kennygilori@gmail.com

Oyeyemi Oluwaseyi Oyebola
(Department of Mathematics and Computer Science, Brandon University, Canada)
E-mail: oyebolaO@brandonu.ca

In this paper, new hyper-algebraic structures called hyperloop, multiloop, polyquasigroup and polyloop, and a special class of polyloop called right Bol polyloop are introduced and studied. It is shown that for any non-commutative (groupoid, quasigroup, loop), commutative and non-commutative (polygroupoid, polyquasigroup, polyloop) can be constructed. It is shown that a right Bol polyloop is characterized by any of seven equivalent identities and has the right alternative properties. Two examples of right Bol loops were constructed with the aid of a ring.

The newly introduced hyper-algebraic structures are:

Definition 1. (Polygroupoid, Polyquasigroup, Polyloop, Multiloop)

Let $\mathcal{M} = (P, \cdot)$ be a polygroupoid. Let $e \in P$ and $\diagup : P \times P \to \mathfrak{P}^*(H)$ and $\diagdown : P \times P \to \mathfrak{P}^*(H)$ such that

(a): (i) $x \in (x \cdot y)\diagup y$ (ii) $x \in (x\diagup y) \cdot y$ (iii) $x \in y\diagdown(y \cdot x)$ (iv) $x \in y \cdot (y\diagdown x)$ for all $x, y \in P$, then $(P, \cdot, \diagdown, \diagup)$ will be called a polyquasigroup.