International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Car+trailers' systems are locally nilpotentizable (a Trieste 2000 conference revisited)

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A car towing a number of passive idealized trailers is a classical kinematical model visualising so-called Goursat distributions. In the description of that series of models (indexed by the number of trailers) there are used trigonometric functions of angles between neighbouring trailers and between the car and its closest trailer. This heavily obscures the algebraic side of the models: the generated control Lie algebra is clearly not nilpotent and infinite-dimensional. Hector Sussmann asked in 1998 if the car+trailers' kinematical systems were nilpotentizable. We presented a positive answer to that question at a Trieste 2000 conference. However, recent scientific meetings show that that our result is not quite known... The aim of the talk is to make better known that result and to sketch our [old] proof of it.

References

[1] Piotr Mormul. Goursat flags: classification of codimension-one singularities. *Journal of Dynamical and Control Systems*, 6(3) : 311–330, 2000.

[2] Piotr Mormul. Minimal nilpotent bases for Goursat distributions of coranks not exceeding six. *Universitatis Iagellonicae Acta Mathematica*, 42 : 15–29, 2004.

Degree theory for proper C^1 Fredholm mappings with applications to boundary value problems on the half line

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We overview elements of the definition and several properties, of a degree theory for proper C^1 Fredholm mappings of index zero [1, 2]. We establish sufficient conditions for solvability of an ODE system $\dot{v} + g(t,w) = f_1(t)$, $\dot{w} + h(t,v) = f_2(t)$ under various boundary conditions on the half line. Note that the unbounded domain prevents the use of Leray-Schauder degree. We establish sufficient conditions for solvability of a semilinear parabolic PDE $u_t - A(t)u + F(t,x,u) = f(t,x)$, once again with conditions at $t = 0$ and as $t \to \infty$. These applications illustrate methods to meet the conditions associated with the degree theory, including smoothness, properness, the Fredholm property, and the establishment of *a priori* bounds. (Note: this is an exposition of work previously published [3, 4].)

References

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How far apart can the projection of the centroid of a convex body and the centroid of its projection be?

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Let K be a convex body in $\mathbb{R}^n$, i.e., a compact convex set with non-empty interior. The centroid (the center of mass) of K is the point

$$c(K) = \frac{1}{|K|} \int_K x\, dx,$$

where $|K|$ denotes the volume of K and the integration is with respect to Lebesgue measure.

In this work we study the following question. Let H be a hyperplane in $\mathbb{R}^n$. Denote by $P_H c(K)$ the orthogonal projection of the centroid of K onto H and by $c(P_H K)$ the centroid of the projection of K onto H. For centrally symmetric bodies these two points coincide, but for non-symmetric bodies these points are generally different. Thus it is natural to ask how far apart these two points can be relative to some linear size of K. More precisely, we are interested in the smallest constant D_n such that for any convex body K in $\mathbb{R}^n$ we have

$$|P_H c(K) - c(P_H K)| \le D_n w_K(u),$$

where u is the unit vector parallel to the segment connecting $P_H c(K)$ and $c(P_H K)$, and $w_K(u)$ is the width of K in the direction of u, given by

$$w_K(u) = \max_{x\in K}\{\langle x, u\rangle\} - \min_{x\in K}\{\langle x, u\rangle\}.$$

Questions of this type began attracting attention several years ago in connection to Grünbaum-type inequalities for sections and projections; see [5], [2]. In particular, an analogue of the question above for sections of convex bodies is stated in [4, p. 127]. For other questions related to distances between various centroids the reader is referred to the book [1, p. 36] and the references contained therein.