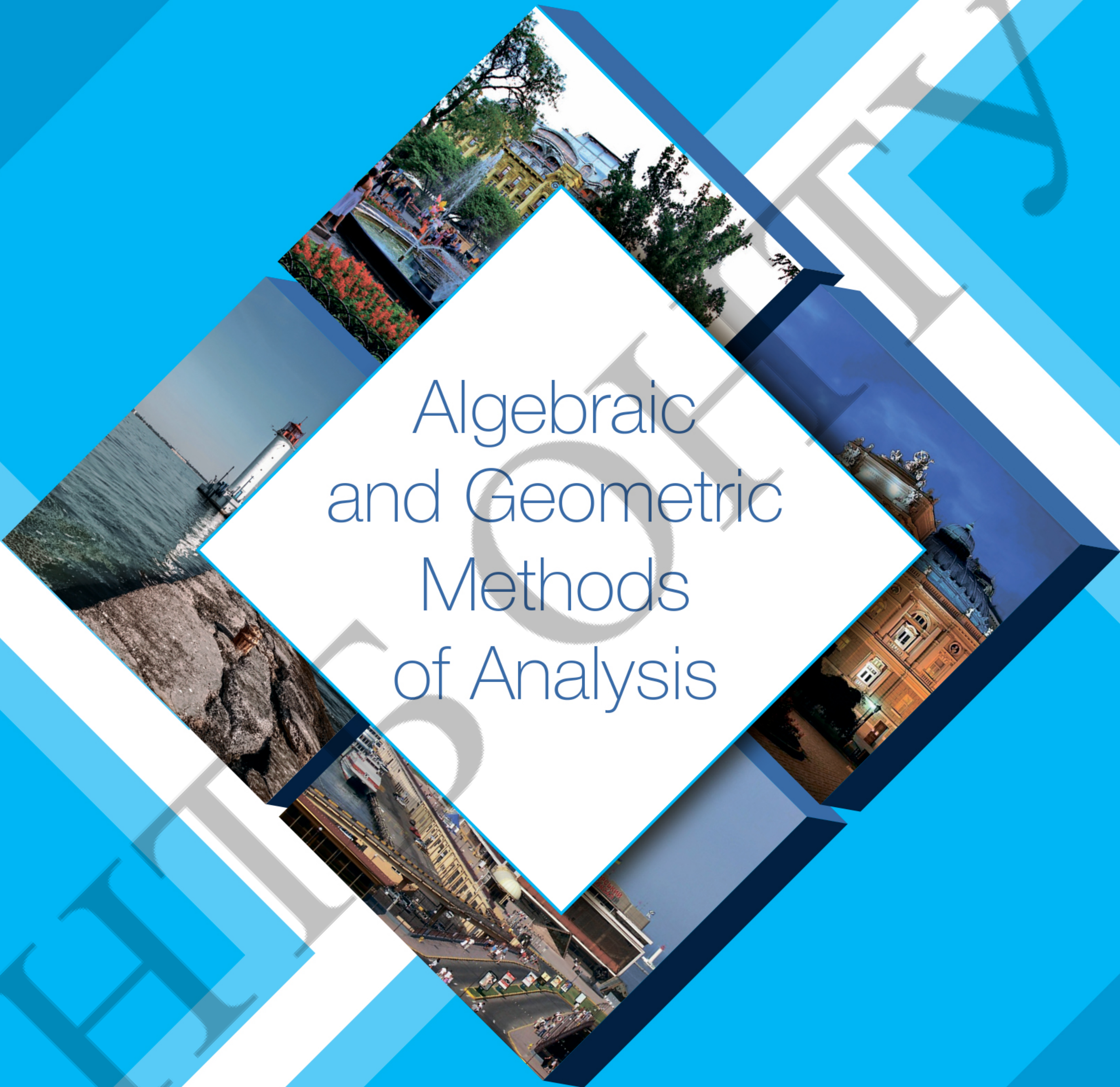
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Periodic point theorem for mappings contracting total pairwise distance

Evgeniy Petrov

(Institute of Applied Mathematics and Mechanics of the NAS of Ukraine, Slovyansk, Ukraine)

E-mail: eugeniy.petrov@gmail.com

We consider a new type of mappings in metric spaces so-called mappings contracting total pairwise distance on n points, see [1]. It is shown that such mappings are continuous. A theorem on the existence of periodic points for such mappings is proved and the classical Banach fixed-point theorem is obtained like a simple corollary as well as the fixed point theorem for mappings contracting perimeters of triangles.

Everywhere below by $|X|$ we denote the cardinality of the set X. Let (X, d) be a metric space, $|X| \geqslant 2$, and let x_1, x_2, ..., $x_n \in X$, $n \geqslant 2$. Denote by

$$S(x_1, x_2, \ldots, x_n) = \sum_{1 \leqslant i < j \leqslant n} d(x_i, x_j)$$

the sum of all pairwise distances between the points from the set $\{x_1, x_2, \ldots, x_n\}$, which we call *total pairwise distance.*

Definition 1. Let $n \geqslant 2$ and let (X, d) be a metric space with $|X| \geqslant n$. We shall say that $T\colon X \to X$ is a *mapping contracting total pairwise distance on n points* if there exists $\alpha \in [0, 1)$ such that the inequality

$$S(Tx_1, Tx_2, \ldots, Tx_n) \leqslant \alpha S(x_1, x_2, \ldots, x_n) \tag{1}$$

holds for all n pairwise distinct points $x_1, x_2, \ldots, x_n \in X$.

Note that the requirement for $x_1, x_2, \ldots, x_n \in X$ to be pairwise distinct is essential, which is confirmed by the following proposition.

Proposition 2. *Suppose that in Definition 1 inequality (1) holds for any n points $x_1, x_2, \ldots, x_n \in X$ with $|\{x_1, x_2, \ldots, x_n\}| = k$, where $2 \leqslant k \leqslant n - 1$. Then T is a mapping contracting total pairwise distance on k points.*

Proposition 3. *Mapping contracting total pairwise distance on m points, $m \geqslant 2$, is a mapping contracting total pairwise distance on n points for all $n > m$.*

Proposition 4. *Mappings contracting total pairwise distance on n points are continuous.*

Let T be a mapping on the metric space X. A point $x \in X$ is called a *periodic point of period* n if $T^n(x) = x$. The least positive integer n for which $T^n(x) = x$ is called the prime period of x. Note that a fixed point is a point of prime period 1.

Theorem 5. *Let $n \geqslant 2$, (X, d) be a complete metric space with $|X| \geqslant n$ and let $T\colon X \to X$ be a mapping contracting total pairwise distance on n points in X. Then T has a periodic point of prime period k, $k \in \{1, \dots, n-1\}$. The number of periodic points is at most $n-1$.*

Let (X, d) be a metric space. Then a mapping $T\colon X \to X$ is called a *contraction mapping* on X if there exists $\alpha \in [0, 1)$ such that

$$d(Tx, Ty) \leqslant \alpha d(x, y) \tag{2}$$

for all $x, y \in X$.

Corollary 6. *(Banach fixed-point theorem) Let (X, d) be a nonempty complete metric space with a contraction mapping $T\colon X \to X$. Then T admits a unique fixed point.*

The following definition was introduced in [2]. In particular, it is a partial case of Definition 1 when $n = 3$.

Definition 7. Let (X, d) be a metric space with $|X| \geqslant 3$. We shall say that $T\colon X \to X$ is a *mapping contracting perimeters of triangles* on X if there exists $\alpha \in [0, 1)$ such that the inequality

$$d(Tx, Ty) + d(Ty, Tz) + d(Tx, Tz) \leqslant \alpha(d(x, y) + d(y, z) + d(x, z))$$

holds for all three pairwise distinct points $x, y, z \in X$.

The following statement was proved in [2,Theorem 2.4] and it is a direct consequence of Theorem 5 in the case $n = 3$.

Corollary 8. *Let (X, d), $|X| \geqslant 3$, be a complete metric space and let $T\colon X \to X$ be a mapping contracting perimeters of triangles on X. Then T has a fixed point if and only if T does not possess periodic points of prime period 2. The number of fixed points is at most two.*

Proposition 9. *Suppose that under the supposition of Theorem 5 the mapping T has a fixed point x^*, which is a limit of some iteration sequence $x_0, x_1 = Tx_0, x_2 = Tx_1, \dots$ such that $x_i \neq x^*$ for all $i = 1, 2, \dots$. Then x^* is the unique fixed point.*

Recall that for a given metric space X, a point $x \in X$ is said to be an *accumulation point* of X if every open ball centered at x contains infinitely many points of X.

Proposition 10. *Let $n \geqslant 2$, (X, d) be a metric space, $|X| \geqslant n$, and let $T\colon X \to X$ be a mapping contracting total pairwise distance on n points. If x is an accumulation point of X, then inequality (2) holds for all points $y \in X$.*

Corollary 11. *Let $n \geqslant 2$, (X, d) be a metric space, $|X| \geqslant n$, and let $T\colon X \to X$ be a mapping contracting total pairwise distance on n points. If all points of X are accumulation points, then T is a contraction mapping.*

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