

International  
Online Conference



**Algebraic  
and Geometric  
Methods of Analysis**

dedicate to the memory  
of Yuriy Trokhymchuk  
(17.03.1928-18.12.2019)

May 25-28, 2021  
Odesa, Ukraine

## LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

## ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

## SCIENTIFIC COMMITTEE

**Drozd Yu.**

*(Kyiv, Ukraine)*

**Maksymenko S.**

*(Kyiv, Ukraine)*

**Plaksa S.**

*(Kyiv, Ukraine)*

**Prishlyak A.**

*(Kyiv, Ukraine)*

**Bakhtin O.**

*(Kyiv, Ukraine)*

**Balan V.**

*(Bucharest, Romania)*

**Banakh T.**

*(Lviv, Ukraine)*

**Borysenko O.**

*(Kharkiv, Ukraine)*

**Cherevko Ye.**

*(Odesa, Ukraine)*

**Fedchenko Yu.**

*(Odesa, Ukraine)*

**Karlova O.**

*(Chernivtsi, Ukraine)*

**Kiosak V.**

*(Odessa, Ukraine)*

**Konovenko N.**

*(Odessa, Ukraine)*

**Lyubashenko V.**

*(Kyiv, Ukraine)*

**Matsumoto K.**

*(Yamagata, Japan)*

**Mormul P.**

*(Warsaw, Poland)*

**Mykhailyuk V.**

*(Chernivtsi, Ukraine)*

**Plachta L.**

*(Krakov, Poland)*

**Pokas S.**

*(Odessa, Ukraine)*

**Sabitov I.**

*(Moscow, Russia)*

**Savchenko O.**

*(Kherson, Ukraine)*

**Sergeeva A.**

*(Odessa, Ukraine)*

**Shelekhov A.**

*(Tver, Russia)*

**Zarichnyi M.**

*(Lviv, Ukraine)*

#### ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Lishchenko N. Dean of faculty of the computer systems and automation ONAFT

#### ORGANIZING COMMITTEE

Cherevko Ye.  
Eftekharinasab K.  
Fedchenko Yu.  
Feshchenko B.  
Khohlyk O.

Klishchuk B.  
Konovenko N.  
Kravchenko A.  
Kuznietsova I.  
Maksymenko S.

Osadchuk E.  
Plakosh A.  
Prus A.  
Sergeeva A.  
Soroka Yu.

# Asymptotically equivalent subspaces of metric spaces

Viktoriia Bilet

(Institute of Applied Mathematics and Mechanics of the NASU, Sloviansk, Ukraine)

*E-mail:* viktoriiabilet@gmail.com

Oleksiy Dovgoshey

(Institute of Applied Mathematics and Mechanics of the NASU, Sloviansk, Ukraine)

*E-mail:* oleksiy.dovgoshey@gmail.com

We investigate the asymptotic behavior of unbounded metric spaces at infinity. To do this we consider a sequence of rescaling metric spaces  $(X, \frac{1}{r_n}d)$  generated by a metric space  $(X, d)$  and a scaling sequence  $(r_n)_{n \in \mathbb{N}}$  of positive reals with  $r_n \rightarrow \infty$ . By definition, the pretangent spaces to  $(X, d)$  at infinity  $\Omega_{\infty, \tilde{r}}^X$  are limit points of this rescaling sequence. We found the necessary and sufficient conditions under which two given unbounded subspaces of  $(X, d)$  have the same pretangent spaces at infinity.

**Definition 1.** Let  $(X, d)$  be an unbounded metric space. Two sequences  $\tilde{x} = (x_n)_{n \in \mathbb{N}} \subset X$  and  $\tilde{y} = (y_n)_{n \in \mathbb{N}} \subset X$  are *mutually stable* with respect to a scaling sequence  $\tilde{r} = (r_n)_{n \in \mathbb{N}}$  if there is a finite limit

$$\lim_{n \rightarrow \infty} \frac{d(x_n, y_n)}{r_n}.$$

For every unbounded metric space  $(X, d)$  and every scaling sequence  $\tilde{r}$ , we denote by  $Seq(X, \tilde{r})$  the set of all sequences  $\tilde{x} = (x_n)_{n \in \mathbb{N}} \subset X$  for which  $\lim_{n \rightarrow \infty} d(x_n, p) = \infty$  and there is a finite limit

$$\lim_{n \rightarrow \infty} \frac{d(x_n, p)}{r_n},$$

where  $p$  is a fixed point of  $X$ .

**Definition 2.** A set  $F \subseteq Seq(X, \tilde{r})$  is *self-stable* if any two  $\tilde{x}, \tilde{y} \in F$  are mutually stable.  $F$  is *maximal self-stable* if it is self-stable and, for arbitrary  $\tilde{y} \in Seq(X, \tilde{r})$ , we have either  $\tilde{y} \in F$  or there is  $\tilde{x} \in F$  such that  $\tilde{x}$  and  $\tilde{y}$  are not mutually stable.

Let  $(X, d)$  be an unbounded metric space, let  $Y$  and  $Z$  be unbounded subspaces of  $X$  and let  $\tilde{r} = (r_n)_{n \in \mathbb{N}}$  be a scaling sequence.

**Definition 3.** The subspaces  $Y$  and  $Z$  are *asymptotically equivalent* with respect to  $\tilde{r}$  if for every

$$\tilde{y}_1 = (y_n^{(1)})_{n \in \mathbb{N}} \in Seq(Y, \tilde{r}) \quad \text{and} \quad \tilde{z}_1 = (z_n^{(1)})_{n \in \mathbb{N}} \in Seq(Z, \tilde{r})$$

there exist

$$\tilde{y}_2 = (y_n^{(2)})_{n \in \mathbb{N}} \in Seq(Y, \tilde{r}) \quad \text{and} \quad \tilde{z}_2 = (z_n^{(2)})_{n \in \mathbb{N}} \in Seq(Z, \tilde{r})$$

such that

$$\lim_{n \rightarrow \infty} \frac{d(y_n^{(1)}, z_n^{(2)})}{r_n} = \lim_{n \rightarrow \infty} \frac{d(y_n^{(2)}, z_n^{(1)})}{r_n} = 0.$$

We shall say that  $Y$  and  $Z$  are *strongly asymptotically equivalent* if  $Y$  and  $Z$  are asymptotically equivalent for all scaling sequences  $\tilde{r}$ .

Let  $(X, d)$  be a metric space and let  $p \in X$ . For every  $t > 0$  we denote by  $S(p, t)$  the sphere with the radius  $t$  and the center  $p$ ,

$$S(p, t) := \{x \in X : d(x, p) = t\},$$

and for every  $Y \subseteq X$  we write

$$S_t^Y := S(p, t) \cap Y.$$

Let  $Y$  and  $Z$  be subspaces of  $(X, d)$ . Define

$$\varepsilon(t, Z, Y) := \sup_{z \in S_t^Z} \inf_{y \in Y} d(z, y)$$

and

$$\varepsilon(t) = \max\{\varepsilon(t, Z, Y), \varepsilon(t, Y, Z)\},$$

where we set  $\varepsilon(t, Z, Y) = 0$  if  $S_t^Z = \emptyset$  and, respectively,  $\varepsilon(t, Y, Z) = 0$  if  $S_t^Y = \emptyset$ .

**Theorem 4.** *Let  $Y$  and  $Z$  be unbounded subspaces of a metric space  $(X, d)$ . Then  $Y$  and  $Z$  are strongly asymptotically equivalent if and only if*

$$\lim_{t \rightarrow \infty} \frac{\varepsilon(t)}{t} = 0.$$

**Corollary 5.** *Let  $(X, d)$  be an unbounded metric space and let  $Y$  be an unbounded subspace of  $X$ . Then the following conditions are equivalent.*

- (1) *For every  $\tilde{r}$  and every maximal self-stable  $\tilde{X}_{\infty, \tilde{r}} \subseteq \text{Seq}(X, \tilde{r})$  there is a maximal self-stable  $\tilde{Y}_{\infty, \tilde{r}} \subseteq \text{Seq}(X, \tilde{r})$  such that  $\tilde{Y}_{\infty, \tilde{r}} \subseteq \tilde{X}_{\infty, \tilde{r}}$  and the embedding  $E_{m_Y} : \Omega_{\infty, \tilde{r}}^Y \rightarrow \Omega_{\infty, \tilde{r}}^X$  is an isometry.*
- (2) *The equality*

$$\lim_{t \rightarrow \infty} \frac{\varepsilon(t, X, Y)}{t} = 0$$

*holds.*

- (3)  *$X$  and  $Y$  are strongly asymptotically equivalent.*

**Remark 6.** Theorem 4 and Corollary 5 can be considered as asymptotic variants of previously proved facts from [1].

## REFERENCES

- [1] Oleksiy Dovgoshey. Tangent spaces to metric spaces and to their subspaces. *Ukr. Mat. Visn.*, 5: 470–487, 2008; Reprinted in *Ukr. Mat. Bull.*, 5(4): 457–477, 2008.

## Зміст

<b>E. Afanas'eva</b> <i>Finitely bi-Lipschitz homeomorphisms between Finsler manifolds</i>	<b>3</b>
<b>Aliyev Yagub</b> <i>About longest and shortest chords passing through a fixed point</i>	<b>5</b>
<b>S. Antonyan</b> <i>Some equivariant properties of Milnor's construction</i>	<b>6</b>
<b>K. Antoshyna, S. Kozerenko</b> <i>Commuting sets for topological set operators</i>	<b>7</b>
<b>B. Apanasov</b> <i>Asymptotic analysis of quasi-regular mappings in space</i>	<b>8</b>
<b>M. J. Atteya</b> <i>Generalized <math>(\sigma, \tau)</math>-derivations on associative rings satisfying certain identities</i>	<b>10</b>
<b>V. Balan</b> <i>The Tucker HO-SVD and the anisotropy of Finslerian geometric models</i>	<b>11</b>
<b>V. Balashchenko, D. Vylegzhanin</b> <i>Invariant structures on homogeneous <math>\Phi</math>-spaces and Lie groups</i>	<b>13</b>
<b>T. Banakh</b> <i>Every 2-dimensional Banach space has the Mazur-Ulam property</i>	<b>15</b>
<b>A. Bandura, V. Baksa, O. Skaskiv</b> <i>A connection between <math>L</math>-index of vector-valued entire function and <math>L</math>-index of each its component</i>	<b>16</b>
<b>B. Baratov, Yu. Eshkabilov</b> <i>Separable cubic stochastic operators</i>	<b>18</b>
<b>V. Bilet, O. Dovgoshey</b> <i>Asymptotically equivalent subspaces of metric spaces</i>	<b>20</b>
<b>E. Bonacci</b> <i>Isomorphic issues about the CTCs in Quantum Physics</i>	<b>22</b>
<b>P. Petrenko, A. Andreev</b> <i>Geometrical Langlands Ramifications and Differential Operators Classification by Verma Module Extensions</i>	<b>23</b>
<b>Y. Cherevko, V. Berezovski, J. Mikeš, Y. Fedchenko</b> <i>Conharmonic Transformations of Locally Conformal Kähler Manifolds</i>	<b>24</b>
<b>V. Chernov</b> <i>Applications of Linking to the Study of Causality</i>	<b>26</b>
<b>A. Bakhtin, I. Denega</b> <i>Problem on extremal decomposition of the complex plane</i>	<b>27</b>
<b>A. Dikarev, A. S. Galaev</b> <i>Parallel spinors on Lorentzian Weyl spaces</i>	<b>29</b>
<b>Yu. A. Drozd</b> <i>Matrix problems, triangulated categories and stable homotopy types</i>	<b>30</b>
<b>V. S. Dryuma</b> <i>On the properties smooth manifolds defined by intersections</i>	<b>31</b>
<b>K. Eftekharinasab</b> <i>Some applications of transversality for infinite dimensional manifolds</i>	<b>33</b>
<b>S. Favorov</b> <i>Uniqueness theorems for almost periodic objects</i>	<b>34</b>
<b>V. Fedorchuk, V. Fedorchuk</b> <i>On symmetry reduction and some classes of invariant solutions of the <math>(1 + 3)</math>-dimensional homogeneous Monge-Ampère equation</i>	<b>35</b>
<b>B. Feshchenko</b> <i>Deformations of circle-valued Morse functions on 2-torus</i>	<b>37</b>