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Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
of Yuriy Trokhymchuk
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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Perfect metrizable of the functor of idempotent measures

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Let \mathbb{R} be the real line. The set $\mathbb{R} \cup \{-\infty\}$ considered with operations: addition \oplus and multiplication \odot defined as $u \oplus v = \max\{u, v\}$ and $u \odot v = u + v$, denotes by \mathbb{R}_{\max} . Let X be a compact Hausdorff space, $C(X)$ the algebra of continuous functions $\varphi: X \rightarrow \mathbb{R}$ with the usual algebraic operations. On $C(X)$ the operations \oplus and \odot we define as $\varphi \oplus \psi = \max\{\varphi, \psi\}$, $\varphi \odot \psi = \varphi + \psi$, $\lambda \odot \varphi = \varphi + \lambda_X$ here $\varphi, \psi \in C(X)$, $\lambda \in \mathbb{R}$. Recall [1] that a functional $\mu: C(X) \rightarrow \mathbb{R}$ is said to be an *idempotent probability measure* on X , if: 1) $\mu(\lambda_X) = \lambda$ for each $\lambda \in \mathbb{R}$; 2) $\mu(\lambda \odot \varphi) = \mu(\varphi) + \lambda$ for all $\lambda \in \mathbb{R}$, $\varphi \in C(X)$; 3) $\mu(\varphi \oplus \psi) = \mu(\varphi) \oplus \mu(\psi)$ for every $\varphi, \psi \in C(X)$. The set of all idempotent probability measures on X we denote by $I(X)$. Consider $I(X)$ as a subspace of $\mathbb{R}^{C(X)}$. The topological space $I(X)$ is compact [1]. For a given map $f: X \rightarrow Y$ of compact Hausdorff spaces the map $I(f): I(X) \rightarrow I(Y)$ defines by the formula $I(f)(\mu)(\varphi) = \mu(\varphi \circ f)$, $\mu \in I(X)$, where $\varphi \in C(Y)$. The construction I is a normal covariant functor, acting in the category of compact Hausdorff spaces and their continuous maps. For $\mu \in I(X)$ we may define the support of $\mu: \text{supp } \mu = \bigcap \{A \subset X : \bar{A} = A, \mu \in I(A)\}$. For a point $x \in X$ by the rule $\delta_x(\varphi) = \varphi(x)$, $\varphi \in C(X)$, we define the Dirac measure δ_x supported on the singleton $\{x\}$.

Put

$$U_S(X) = \left\{ \lambda: X \rightarrow [-\infty, 0] \mid \lambda \text{ is upper semicontinuous and there exists a } x_0 \in X \text{ such that } \lambda(x_0) = 0 \right\}.$$

Then we have

$$I(X) = \left\{ \bigoplus_{x \in X} \lambda(x) \odot \delta_x : \lambda \in U_S(X) \right\}.$$

We define a subset

$$I_\omega(X) = \left\{ \bigoplus_{x \in X} \lambda(x) \odot \delta_x : \lambda \in U_S(X), |\{x \in X : \lambda(x) > -\infty\}| < \infty \right\} \subset I(X).$$

$I_\omega(X)$ is everywhere dense in $I(X)$ [1, 2]. Put

$$\rho_2(\mu_1, \mu_2) = \inf \left\{ \frac{\sum_{(x,y) \in \text{supp } \xi} e^{\lambda_1(x) + \lambda_2(y)} \cdot \rho(x, y)}{\sum_{x \in \text{supp } \mu_1} e^{\lambda_1(x)} \cdot \sum_{y \in \text{supp } \mu_2} e^{\lambda_2(y)}} : \xi \in \Lambda_{12} \right\},$$

where $\mu_i = \bigoplus_{x \in X} \lambda_i(x) \odot \delta_x \in I_\omega(X)$, $i = 1, 2$. Further, for every pair $\mu, \nu \in I(X)$ take consequences $\{\mu_n\}, \{\nu_n\} \subset I_\omega(X)$ such that $\lim_{n \rightarrow \infty} \mu_n = \mu$ and $\lim_{n \rightarrow \infty} \nu_n = \nu$, and put

$$\rho_I(\mu, \nu) = \lim_{n \rightarrow \infty} \rho_2(\mu_n, \nu_n).$$

The function ρ_I is a metric on $I(X)$ generating the pointwise convergence topology on $I(X)$ and the restriction of which coincides with the metric ρ on X .

Consider a system ψ consisting of all maps $\psi_X: I^2(X) \rightarrow I(X)$, acting as the following. Given $M \in I^2(X)$ put $\psi_X(M)(\varphi) = M(\bar{\varphi})$, where for any function $\varphi \in C(X)$ the function $\bar{\varphi}: I(X) \rightarrow \mathbb{R}$ defines by the formula $\bar{\varphi}(\mu) = \mu(\varphi)$. Fix a compactum X and for a positive integer n put $\psi_{n+1, n} = \psi_{I^{n-1}(X)}: I^{n+1}(X) \rightarrow I^n(X)$. Note that $\psi_{n+1, n} \circ \eta_{n, n+1} = Id_{I^n(X)}$.

Lemma 1. $\psi_{1,0}: (I^2(X), \rho_{I^2}) \rightarrow (I(X), \rho_I)$ is a non-expanding map.

Lemma 2. For each $N \in \psi_{1,0}^{-1}(\mu)$ we have $\rho_I(\mu, \delta_{x_0}) = \rho_{I^2}(\delta_{\delta_{x_0}}, N)$.

Lemma 3. If $\rho_I(\mu, \eta_{0,1}(X)) \geq \varepsilon$ then $\rho_{I^2}(I(\eta_{0,1})(\mu), \eta_{1,2}(I(X))) \geq \varepsilon$.

Theorem 4. The functor I is perfect metrizable.

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