International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

Scientific Committee

- **Bolotov D.** (*Kharkiv, Ukraine*)
- **Bondarenko V.** (*Kyiv, Ukraine*)
- **Boychuk O.** (*Kyiv, Ukraine*)
- **Boyko V.** (*Kyiv, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Dorogovtsev A.** (*Kyiv, Ukraine*)
- **Drozd Yu.** (*Kyiv, Ukraine*)
- **Gerasymenko V.** (*Kyiv, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Kiosak V.** (*Odesa, Ukraine*)
- **Kochubei A.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Mikhailets V.** (*Kyiv, Ukraine*)
- **Ostrovskyi V.** (*Kyiv, Ukraine*)
- **Petravchuk A.** (*Kyiv, Ukraine*)
- **Plaksa S.** (*Kyiv, Ukraine*)
- **Portenko M.** (*Kyiv, Ukraine*)
- **Pratsiovytyi M.** (*Kyiv, Ukraine*)
- **Savchenko O.** (*Kherson, Ukraine*)
- **Romanyuk A.** (*Kyiv, Ukraine*)
- **Timokha O.** (*Kyiv, Ukraine*)

Organizing commitee

- **Maksymenko S.** (*Kyiv, Ukraine*)
- **Konovenko N.** (*Odesa, Ukraine*)
- **Fedchenko Yu.** (*Odesa, Ukraine*)
- **Cherevko Ye.** (*Odesa, Ukraine*)
- **Osadchuk Ye.** (*Odesa, Ukraine*)
- **Sergeeva O.** (*Odesa, Ukraine*)

Fuzzy metrization of spaces of ⋆-measures

Aleksandr Savchenko
(Kherson State University, Universyteska st., 27, Kherson, 73003, Ukraine)
E-mail: `savchenko.o.g@ukr.net`

In [1], a fuzzy metrization of spaces if idempotent measures is constructed. The idempotent measures are known to be counterparts of the probability measures in the idempotent mathematics (see [4] for detailed exposition of topological aspects of idempotent measures).

Definition 1. A binary operation $*\colon [0,1]\times[0,1]\to[0,1]$ is a *continuous t-norm* if it satisfies the following conditions.

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a*1=a$ for all $a\in[0,1]$,
(4) $a*b\le c*d$ whenever $a\le c$ and $b\le d$, for each $a,b,c,d\in[0,1]$.

Definition 2. A 3-tuple $(X,M,*)$ is said to be a *fuzzy metric space* [2] if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X\times X\times(0,\infty)$ satisfying the following conditions for all $x,y,z\in X$ and $s,t\in(0,\infty)$:

(1) $M(x,y,t)>0$;
(2) $M(x,y,t)=1$ if and only if $x=y$;
(3) $M(x,y,t)=M(y,x,t)$;
(4) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$;
(5) the function $M(x,y,\cdot)\colon(0,\infty)\to[0,1]$ is continuous.

If $(X,M,*)$ is a fuzzy metric space, $(M,*)$ will be called a fuzzy metric on X.

Let $\star$ be a triangular norm. A functional $\mu\colon C(X,[0,1])\to[0,1]$ is said to be an $\star$-measure if the following holds (c_X is the constant function with value c):

(1) $\mu(c_X)=c$ for all $c\in[0,1]$;
(2) $\mu(\varphi\oplus\psi)=\mu(\varphi)\oplus\mu(\psi)$;
(3) $\mu(\lambda\star\varphi)=\lambda\star\mu(\varphi)$.

(Here, $\oplus$ denotes the max operation.)

The spaces $I^\star(X)$ of $\star$-measures on compact Hausdorff spaces X are endowed with the weak* topology [3].

The aim of the talk is to provide a fuzzy metrization of the spaces $I^\star(X)$ on fuzzy metric spaces $(X,M,*)$. To this end, we identify the spaces $I^\star(X)$ with subsets of the hyperspace of $X\times[0,1]$. Our results are analogs of those in [1].

References

[1] V. Brydun, A. Savchenko, M. Zarichnyi, Fuzzy metrization of the spaces of idempotent measures, European Journal of Mathematics. 2020. V. 6. N 1. 98–109. DOI 10.1007/s40879-019-00341-8.
[2] A. George, P. Veeramani, On some result in fuzzy metric space. Fuzzy Sets Syst. 64, 395-399, (1994)
[3] Kh. Sukhorukova, Spaces of non-additive measures generated by triangular norms, Preprint.
[4] M. Zarichnyi, *Spaces and mappings of idempotent measures*. Izvestiya: Math. 2010, 74 (3), 481⊠-499. doi: 10.4213/im2785