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ФІТБ ОНАФТ

## Lebesgue-type inequalities for the Fourier sums

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Let  $L_p$ ,  $1 \leq p < \infty$ , be the space of  $2\pi$ -periodic functions  $f$  summable to the power  $p$  on  $[0, 2\pi)$ , in which the norm is given by the formula  $\|f\|_p = \left( \int_0^{2\pi} |f(t)|^p dt \right)^{\frac{1}{p}}$ ; and  $C$  be the space of continuous  $2\pi$ -periodic functions  $f$ , in which the norm is specified by the equality  $\|f\|_C = \max_t |f(t)|$ .

Denote by  $C_\beta^{\alpha,r} L_p$ ,  $\alpha > 0$ ,  $r > 0$ ,  $\beta \in \mathbb{R}$ ,  $1 \leq p \leq \infty$ , the set of all  $2\pi$ -periodic functions, representable for all  $x \in \mathbb{R}$  as convolutions of the form (see, e.g., [1, p. 133])

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} P_{\alpha,r,\beta}(x-t) \varphi(t) dt, \quad a_0 \in \mathbb{R}, \quad \varphi \perp 1, \quad \varphi \in L_p, \quad (1)$$

where  $P_{\alpha,r,\beta}(t)$  are generalized Poisson kernels

$$P_{\alpha,r,\beta}(t) = \sum_{k=1}^{\infty} e^{-\alpha k r} \cos \left( kt - \frac{\beta \pi}{2} \right), \quad \alpha, r > 0, \quad \beta \in \mathbb{R}.$$

If the functions  $f$  and  $\varphi$  are related by the equality (1), then function  $f$  in this equality is called generalized Poisson integral of the function  $\varphi$ . The function  $\varphi$  in equality (1) is called as generalised derivative of the function  $f$  and is denoted by  $f_\beta^{\alpha,r}$ .

Let  $E_n(f)_{L_p}$  be the best approximation of the function  $f \in L_p$  in the metric of space  $L_p$ ,  $1 \leq p \leq \infty$  by the trigonometric polynomials  $t_{n-1}$  of degree  $n-1$ , i.e.,

$$E_n(f)_{L_p} = \inf_{t_{n-1}} \|f - t_{n-1}\|_{L_p}.$$

Our aim is to obtain of asymptotically best possible Lebesgue-type inequalities, for functions from the class  $C_\beta^{\alpha,r} L_p$ , where norms  $\|f(\cdot) - S_{n-1}(f; \cdot)\|_C$  are estimated via best approximations  $E_n(f_\beta^{\alpha,r})_{L_p}$  for  $0 < r < 1$  and  $1 \leq p < \infty$ . Here  $S_{n-1}(f; \cdot)$  is the partial Fourier sums of order  $n-1$  for a function  $f$ . For  $r \geq 1$  such inequalities were established in [1]–[3].

For arbitrary  $\alpha > 0$ ,  $r \in (0, 1)$  and  $1 \leq p < \infty$  we denote by  $n_0 = n_0(\alpha, r, p)$  the smallest integer  $n$  such that

$$\frac{1}{\alpha r} \frac{1}{n^r} + \frac{\alpha r p}{n^{1-r}} \leq \begin{cases} \frac{1}{14}, & p = 1, \\ \frac{1}{(3\pi)^3} \cdot \frac{p-1}{p}, & 1 < p < \infty. \end{cases}$$

We use Gauss hypergeometric function  $F(a, b; c; d)$  of the form

$$F(a, b; c; z) = 1 + \sum_{k=1}^{\infty} \frac{(a)_k (b)_k}{(c)_k} \frac{z^k}{k!},$$

where  $(x)_k$  is the Pochhammer symbol, defined by  $(x)_k := x(x+1)\dots(x+k-1)$ .

We showed that the following theorems hold.

**Theorem 1.** Let  $0 < r < 1$ ,  $\alpha > 0$ ,  $\beta \in \mathbb{R}$ ,  $1 < p < \infty$ , and  $n \in \mathbb{N}$ . Then for any function  $f \in C_{\beta}^{\alpha,r} L_p$  and  $n \geq n_0(\alpha, r, p)$ , the following inequality is true:

$$\begin{aligned} \|f(\cdot) - S_{n-1}(f; \cdot)\|_C &\leq e^{-\alpha n^r} n^{\frac{1-r}{p}} \left( \frac{\|\cos t\|_{p'}}{\pi^{1+\frac{1}{p'}} (\alpha r)^{\frac{1}{p}}} F^{\frac{1}{p'}} \left( \frac{1}{2}, \frac{3-p'}{2}; \frac{3}{2}; 1 \right) + \right. \\ &+ \gamma_{n,p} \left( \left( 1 + \frac{(\alpha r)^{\frac{p'-1}{p}}}{p'-1} \right) \frac{1}{n^{\frac{1-r}{p}}} + \frac{(p)^{\frac{1}{p'}}}{(\alpha r)^{1+\frac{1}{p}}} \frac{1}{n^r} \right) \left. E_n(f_{\beta}^{\alpha,r})_{L_p}, \quad \frac{1}{p} + \frac{1}{p'} = 1, \right. \end{aligned} \quad (2)$$

where  $F(a, b; c; d)$  is Gauss hypergeometric function.

Moreover for any function  $f \in C_{\beta}^{\alpha,r} L_p$  one can find a function  $F(x) = F(f; p; n; x)$ , such that  $E_n(F_{\beta}^{\alpha,r})_{L_p} = E_n(f_{\beta}^{\alpha,r})_{L_p}$  and for  $n \geq n_0(\alpha, r, p)$  the following equality is true

$$\begin{aligned} \|F(\cdot) - S_{n-1}(F; \cdot)\|_C &= e^{-\alpha n^r} n^{\frac{1-r}{p}} \left( \frac{\|\cos t\|_{p'}}{\pi^{1+\frac{1}{p'}} (\alpha r)^{\frac{1}{p}}} F^{\frac{1}{p'}} \left( \frac{1}{2}, \frac{3-p'}{2}; \frac{3}{2}; 1 \right) + \right. \\ &+ \gamma_{n,p} \left( \left( 1 + \frac{(\alpha r)^{\frac{p'-1}{p}}}{p'-1} \right) \frac{1}{n^{\frac{1-r}{p}}} + \frac{(p)^{\frac{1}{p'}}}{(\alpha r)^{1+\frac{1}{p}}} \frac{1}{n^r} \right) \left. E_n(f_{\beta}^{\alpha,r})_{L_p}, \quad \frac{1}{p} + \frac{1}{p'} = 1. \right. \end{aligned} \quad (3)$$

In (2) and (3) the quantity  $\gamma_{n,p} = \gamma_{n,p}(\alpha, r, \beta)$  is such that  $|\gamma_{n,p}| \leq (14\pi)^2$ .

**Theorem 2.** Let  $0 < r < 1$ ,  $\alpha > 0$ ,  $\beta \in \mathbb{R}$ ,  $n \in \mathbb{N}$ . Then, for any  $f \in C_{\beta}^{\alpha,r} L_1$  and  $n \geq n_0(\alpha, r, 1)$ , the following inequality holds:

$$\|f(\cdot) - S_{n-1}(f; \cdot)\|_C \leq e^{-\alpha n^r} n^{1-r} \left( \frac{1}{\pi \alpha r} + \gamma_{n,1} \left( \frac{1}{(\alpha r)^2} \frac{1}{n^r} + \frac{1}{n^{1-r}} \right) \right) E_n(f_{\beta}^{\alpha,r})_{L_1}. \quad (4)$$

Moreover for any function  $f \in C_{\beta}^{\alpha,r} L_1$  one can find a function  $F(x) = F(f; n, x)$  in the set  $C_{\beta}^{\alpha,r} L_1$ , such that  $E_n(F_{\beta}^{\alpha,r})_{L_1} = E_n(f_{\beta}^{\alpha,r})_{L_1}$  and for  $n \geq n_0(\alpha, r, 1)$  the following equality holds

$$\|F(\cdot) - S_{n-1}(F; \cdot)\|_C = e^{-\alpha n^r} n^{1-r} \left( \frac{1}{\pi \alpha r} + \gamma_{n,1} \left( \frac{1}{(\alpha r)^2} \frac{1}{n^r} + \frac{1}{n^{1-r}} \right) \right) E_n(f_{\beta}^{\alpha,r})_{L_1}. \quad (5)$$

In (4) and (5), the quantity  $\gamma_{n,p} = \gamma_{n,p}(\alpha, r, \beta)$  is such that  $|\gamma_{n,p}| \leq (14\pi)^2$ .

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