International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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[4] M. Breuer, *Fredholm theories in von Neumann algebras. II*, Math. Ann. **180**, 313–325 (1969). https://doi.org/10.1007/BF01351884

[5] E. I. Fredholm, *Sur une classe d'equations fontionnelles*, Acta Math. **27** (1903), 365–390.

[6] S. Ivković, *Semi-Fredholm theory on Hilbert C*-modules,* Banach J. Math. Anal., **13** (4) , 989-1016 October (2019) doi:10.1215/17358787-2019-0022. https://projecteuclid.org/euclid.bjma/1570608171

[7] S. Ivković, *On operators with closed range and semi-Fredholm operators over W*-algebras*, Russ. J. Math. Phys. **27**, 48-60 (2020) http://link.springer.com/article/10.1134/S1061920820010057

[8] S. Ivković, *On various generalizations of semi-A-Fredholm operators*, Complex Anal. Oper. Theory **14**, 41 (2020). https://doi.org/10.1007/s11785-020-00995-3

[9] S. Ivković, *On Upper Triangular Operator* 2×2 *Matrices Over C*-Algebras*, FILOMAT, (2020), vol. **34** no. 3, 691-706. https://doi.org/10.2298/FIL2003691I

[10] S. Ivković, *On Drazin invertible C*-operators and generalized C*-Weyl operators*, Ann. Funct. Anal. **14**, 36 (2023). https://doi.org/10.1007/s43034-023-00258-0

[11] S. Ivković, *Semi-Fredholm theory in C*-algebras*, https://arxiv.org/abs/2002.04905

[12] D. J. Kečkić, Z. Lazović, *Fredholm operators on C*-algebras.* ActaSci.Math. **83** , 629-655 (2017). https://doi.org/10.14232/actasm-015-526-5

[13] A. S. Mishchenko, A.T. Fomenko, *The index of eliptic operators over C*-algebras*, Izv. Akad. Nauk SSSR Ser. Mat. **43** (1979), 831–859; English transl., Math. USSR-Izv.**15** (1980) 87–112.

[14] S. C. Živković Zlatanović, *An Introduction into Fredholm Theory and Generalized Drazin-Riesz Invertible Operators*, **20** (28), pp. 113–198, Matematički institut SANU, Beograd (2022). ISSN: 0351-9406

On some non-associative hyper-algebraic structures

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In this paper, new hyper-algebraic structures called hyperloop, multiloop, polyquasigroup and polyloop, and a special class of polyloop called right Bol polyloop are introduced and studied. It is shown that for any non-commutative (groupoid, quasigroup, loop), commutative and non-commutative (polygroupoid, polyquasigroup, polyloop) can be constructed. It is shown that a right Bol polyloop is characterized by any of seven equivalent identities and has the right alternative properties. Two examples of right Bol loops were constructed with the aid of a ring.

The newly introduced hyper-algebraic structures are:

Definition 1. (Polygroupoid, Polyquasigroup, Polyloop, Multiloop)

Let $\mathcal{M} = (P, \cdot)$ be a polygroupoid. Let $e \in P$ and $\diagup : P \times P \to \mathfrak{P}^*(H)$ and $\diagdown : P \times P \to \mathfrak{P}^*(H)$ such that

(a): (i) $x \in (x \cdot y)\diagup y$ (ii) $x \in (x\diagup y) \cdot y$ (iii) $x \in y\diagdown(y \cdot x)$ (iv) $x \in y \cdot (y\diagdown x)$ for all $x, y \in P$, then $(P, \cdot, \diagdown, \diagup)$ will be called a polyquasigroup.

(b): $x \cdot e = e \cdot x = x$ for all $x \in P$ and $(P, \cdot, \diagdown, \diagup)$ is a polyquasigroup. Then $(P, \cdot, \diagdown, \diagup, e)$ will be called a polyloop.

(c): $x \in x \cdot e = e \cdot x$ for all $x \in P$ and $(P, \cdot, \diagdown, \diagup)$ is a polyquasigroup. Then $(P, \cdot, \diagdown, \diagup, e)$ will be called a multiloop.

(d): $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x, y, z \in P$ and $(P, \cdot, \diagdown, \diagup)$ is a polyloop. Then $(P, \cdot, \diagdown, \diagup)$ will be called an associative polyloop.

Definition 2. (Right Bol Polyloop)
Let $\mathcal{M} = (P, \cdot, \diagdown, \diagup, e)$ be a polyloop, then $(P, \cdot, \diagdown, \diagup, e)$ will be called a right Bol Polyloop, if it satisfies the identity

$$(xy \cdot z)y = x(yz \cdot y) \ \forall \ x, y, z \in P \tag{1}$$

Result on equivalence between the hyper-algebraic structures in Definition 1 and some existing ones in literature is presented in Theorem 3.

Theorem 3. *Let $(G, \cdot)$ be a polygroupoid.*

(1) *The following are equivalent:*
   (a) *$(G, \cdot)$ is an hyperquasigroup.*
   (b) *$(G, \cdot, \diagdown, \diagup)$ is a polyquasigroup.*
   (c) *$(G, \cdot)$ is an quasigrouphypergroup.*
   (d) *There exist hyperoperations $\diagdown$ and $\diagup$ on G such that $z \in x \cdot y \Leftrightarrow x \in z \diagup y \Leftrightarrow y \in x \diagdown z$ holds for all $x, y, z \in G$.*

(2) *$(G, \cdot, e)$ is a hyperloop if and only if it $(G, \cdot, e)$ is a multiloop.*
(3) *$(G, \cdot)$ is a hypergroup if and only if it is an associative polyquasigroup.*
(4) *$(G, \cdot)$ is an H_v-group if and only if it is a polyquasigroup with WASS.*
(5) *$(G, \cdot)$ is a Marty-Moufang hypergroup (H_m-group) if and only if it is a Moufang polyquasigroup. (Marty-Moufang hypergroup of Bayon and Lygeros [1])*
(6) *$(G, \cdot)$ is a polygroup if and only if it is a associative polyloop.*

Theorem 4 describes a method of construction of commutative and non-commutative polyquasigroups (polyloops) using a non-commutative quasigroup (loop).

Theorem 4. *(Construction of polygroupoid, polyquasigroup and polyloop)*
Given a non-commutative groupoid (quasigroup, loop) $(G, \cdot, \diagdown, \diagup, e)$, define an hyperoperation $\odot : G \times G \to \mathfrak{P}^*(G)$ as $x \odot y = \{xy, yx\}$. Then, there exist left division and right division hyperoperations $\leftthreetimes : G \times G \to \mathfrak{P}^*(G)$ and $\rightthreetimes : G \times G \to \mathfrak{P}^*(G)$ of $\odot$ such that $x \leftthreetimes y = \{x \diagdown y, y \diagup x\}$ and $x \rightthreetimes y = \{x \diagup y, y \diagdown x\}$ respectively and

(1) *$(G, \odot)$ is a commutative polygroupoid.*
(2) *$(G, \odot, \leftthreetimes, \rightthreetimes)$ is a commutative polyquasigroup while $(G, \leftthreetimes, \odot, \leftthreetimes)$ and $(G, \rightthreetimes, \rightthreetimes, \odot)$ are non-commutative polyquasigroups.*
(3) *$(G, \odot, \leftthreetimes, \rightthreetimes, e)$ is a commutative polyloop while $(G, \leftthreetimes, \odot, \leftthreetimes)$ and $(G, \rightthreetimes, \rightthreetimes, \odot)$ are non-commutative polyquasigroups.*

Theorem 5 presents some results on the algebraic properties and characterization of right Bol polyloop as defined by (1) of Definition 2.

Theorem 5. *Let $(P, \cdot, \diagdown, \diagup, e)$ be a polyloop. Then $(P, \cdot, \diagdown, \diagup, e)$ is a right Bol polyloop if and only if either of the following conditions holds:*

(i) $X(yz \cdot y) = (Xy \cdot z)y$

(ii) $x(yZ \cdot y) = (xy \cdot Z)y$
(iii) $x(Yz \cdot Y) = (xY \cdot z)Y$
(iv) $X(yZ \cdot y) = (Xy \cdot Z)y$
(v) $X(Yz \cdot Y) = (XY \cdot z)Y$ *(vi)* $x(YZ \cdot Y) = (xY \cdot Z)Y$
(vi) $X(YZ \cdot Y) = (XY \cdot Z)Y$ *for all* $x, y, z \in P$ *and* $X, Y, Z \subseteq P$.

Example 6. Let $(\mathbb{Z}_2, +, \cdot)$ be the ring of integer modulo 2 and let $G = \mathbb{Z}_2^3$. For (i, j, k) and (p, q, r) in G, define

$$(i, j, k) * (p, q, r) = (i + p, j + q, k + r + jpq).$$

Consider $\mathbb{Z}_2^3//N \subseteq P(\mathbb{Z}_2^3)$ where $N = N(\mathbb{Z}_2^3, *) = \{(0,0,0), (0,1,0), (1,0,0), (0,1,1)\}$ is the nucleus of $(\mathbb{Z}_2^3, *)$ so that

$$\mathbb{Z}_2^3//N = \Big\{ \big\{ (i,j,k), (i,j+1,k), (i,j,k+1), (i+1,j,k), (i,j+1,k+1) \big\} \ \Big| \ i,j,k \in \mathbb{Z}_2 \Big\}.$$

Define an hyperoperation $'\circ'$ on $\mathbb{Z}_2^3//N$ as follows

$$(i,j,k)N \circ (p,q,r)N = \Big\{ \Big\{ \big(i+a+p, j+b+q, k+c+jab+r+(j+b)pq\big), \big(i+a+p, j+b+q+1, k+c+jab+r+(j+b)pq\big), \big(i+a+p, j+b+q, k+c+jab+r+(j+b)pq+1\big), \big(i+a+p+1, j+b+q, k+c+jab+k+(j+b)pq\big), \big(i+a+p, j+b+q+1, k+c+jab+r+(j+b)pq+1\big) \Big\} \ \Big| \ i,j,k,p,q,r \in \mathbb{Z}_2, a,b,c \in N \Big\}.$$

Then, $\big(\mathbb{Z}_2^3//N, \circ\big)$ is a right Bol polyloop.

References

[1] Bayon R. and Lygeros N. The H_v-groups and Marty-Moufang hypergroups. *Proceedings of the 1st International Conference on Algebraic Informatics*, Aristotle Univ. Thessaloniki, Thessaloniki, 285–294, 2005.

[2] Oyebola, O.O. and Jaiyéọlá, T. G. Non-associative algebraic hyperstructures and their applications to biological inheritance. *Monografías Matemáticas García de Galdeano* 42: 229–-241, 2019.

The rank of Mordell-Weil groups of surfaces

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Let $S \longrightarrow C$ be a fibraton of surface, and we can define Mordell-Weil groups. In fact, they are Abelian groups. In 1989, Prof. Mok raised the following question in [1]:

Problem 1. How to determine the rank of Mordell-Weil group >0?

In [2] and [4], the authors discuss the above problem. In this talk, we try to give some new views in this problem. Especially, we use the number of singular fibers to determine whether the rank is zero or not.

Theorem 2. *Let* $S \to \mathbb{P}^1$ *be a fibraton of surface. If* $s_1 > 4g$*, then the rank of Mordell-Weil group* > 0*, where* s_1 *is the number of fiber whose Jacobian is singular.*