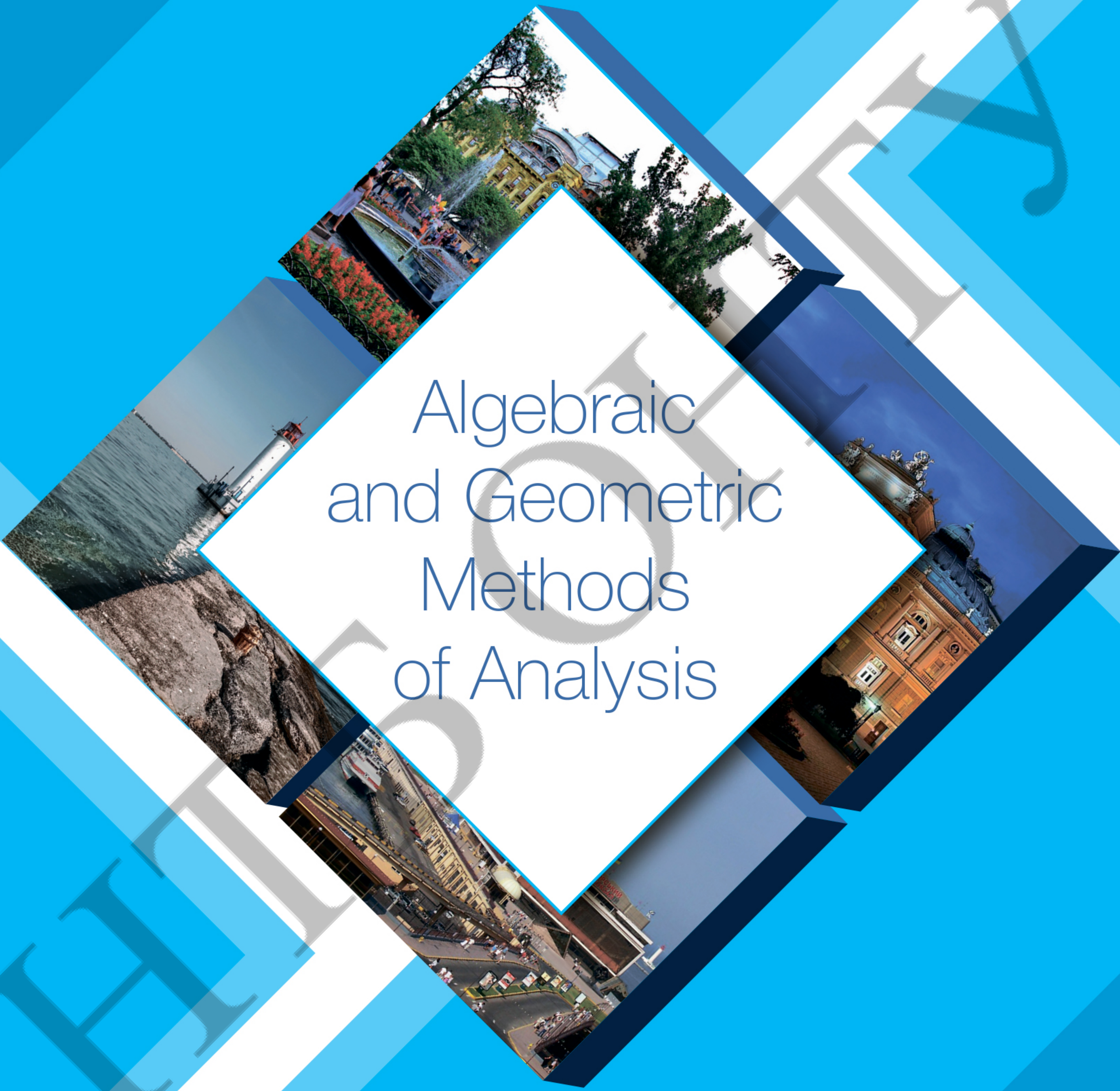
International
Scientific Conference



Algebraic and Geometric Methods of Analysis

27-30 May 2024

Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Approximation by interpolation trigonometric polynomials on the sets of infinitely differentiable functions

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Denote by $C^{\psi}_{\beta}L_1$ the set of 2π–periodic functions, which for all $x \in \mathbb{R}$ can be represented as convolutions of the form

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi}\int\limits_{-\pi}^{\pi} \Psi_\beta(x-t)\varphi(t)dt, \ a_0 \in \mathbb{R}, \ \varphi \in L_1, \ \varphi \perp 1 \tag{1}$$

with the generating kernel Ψ_β of the form

$$\Psi_\beta(t) = \sum_{k=1}^{\infty} \psi(k)\cos\left(kt - \frac{\beta\pi}{2}\right), \ \psi(k) > 0, \ \beta \in \mathbb{R},$$

such that

$$\sum_{k=1}^{\infty} \psi(k) < \infty.$$

The function φ in equality (1) is called as (ψ,β)–derivative of the function f and is denoted by f^{ψ}_{β} $(\varphi(\cdot) = f^{\psi}_{\beta}(\cdot))$ [1].

We study approximation properties of the sets $C^{\psi}_{\beta}L_1$, where we use as approximation aggregate the classical interpolation trigonometric Lagrange polynomials, which are defined by odd number of uniformly distributed interpolation nodes.

For arbitrary function $f(x)$ from C by $\widetilde{S}_{n-1}(f;x)$, $n \in \mathbb{N}$, we will denote the trigonometric polynomial of the order $n-1$, which interpolates $f(x)$ in the nodes $x_k^{(n-1)} = \frac{2k\pi}{2n-1}$, $k \in \mathbb{Z}$, namely, such that

$$\tilde{S}_{n-1}(f; x_k^{(n-1)}) = f(x_k^{(n-1)}), \ k = 0, 1, ..., 2n-2.$$

The polynomial $\widetilde{S}_{n-1}(f;\cdot)$ is unequivocally defined by mentioned interpolation conditions, is called as Lagrange interpolation polynomial and can be represented in the explicit form via Dirichlet kernel

$$D_{n-1}(t) = \frac{1}{2} + \sum_{k=1}^{n-1}\cos kt = \frac{\sin(n-\frac{1}{2})t}{2\sin\frac{t}{2}}$$

in the following way

$$\tilde{S}_{n-1}(f;x) = \frac{2}{2n-1}\sum_{k=0}^{2n-2} f(x_k^{(n-1)})D_{n-1}(x - x_k^{(n-1)}).$$

Let $\mathcal{T}_{2n-1}$ be the space of all trigonometric polynomials of degree at most $n-1$ and let $E_n(f)_{L_1}$ be the best approximation of the function $f \in L_1$ in the metric of space L_1, by the trigonometric polynomials t_{n-1} of degree $n-1$, i.e.,

$$E_n(f)_{L_1} = \inf_{t_{n-1}\in\tau_{2n-1}} \|f - t_{n-1}\|_{L_1}.$$

Denote by $\tilde{\rho}_n(f;\cdot)$ the deviation of the function $f \in C$ from its interpolation Lagrange polynomial $\tilde{S}_{n-1}(f;\cdot)$

$$\tilde{\rho}_n(f;x) = f(x) - \tilde{S}_{n-1}(f;x).$$

Let

$$\mathfrak{M}=\left\{\psi\in C[1,\infty)\colon \psi(t)>0, \psi(t_1 - 2\psi((t_1+t_2)/2) + \psi(t_2) \geq 0 \ \forall t_1, t_2 \in [1,\infty),\ \lim_{t\to\infty}\psi(t)=0\right\}.$$

We consider for each function $\psi \in \mathfrak{M}$ the following characteristics

$$\alpha(t) = \alpha(\psi;t) := \frac{\psi(t)}{t|\psi'(t)|}, \quad \psi'(t) := \psi'(t+0),$$

$$\lambda(t) = \lambda(\psi;t) := \frac{\psi(t)}{|\psi'(t)|}.$$

As it was shown in [2], if $\alpha(t) \downarrow 0$ as $t \to \infty$, then the sets $C^{\psi}_{\beta}L_1$ are the sets of infinitely differentiable functions.

Our aim is to establish the asymptotically best possible interpolation analogues of the Lebesgue type inequalities for the functions f from the sets $C^{\psi}_{\beta}L_1$, $\beta \in \mathbb{R}$, where the upper estimates of the quantities $|\tilde{\rho}_n(f;x)|$, $x \in \mathbb{R}$, are expressed via the best approximations $E_n(f^{\psi}_{\beta})_{L_1}$.

The following theorem takes place.

Theorem 1. *Let $\psi \in \mathfrak{M}$ and characteristics $\alpha(t)$ and $\lambda(t)$ satisfy the conditions*

$$\alpha(t) \downarrow 0, \quad t \to \infty,$$

$$\lambda(t) \uparrow \infty, \quad t \to \infty.$$

Then, for arbitrary function $f \in C^{\psi}_{\beta}L_1$, $\beta \in \mathbb{R}$, in every point $x \in \mathbb{R}$ for all $n \in \mathbb{N}$ such that

$$\alpha(n) \leq \frac{1}{4},$$

the following inequality takes place

$$|\tilde{\rho}_n(f;x)| \leq \frac{2}{\pi}\left|\sin\frac{2n-1}{2}x\right| \psi(n)\lambda(n)\left(1 + 4\alpha(n) + \frac{1}{\lambda(n)}\right) E_n(f^{\psi}_{\beta})_{L_1}.$$

Moreover for arbitrary function $f \in C^{\psi}_{\beta}L_1$ one can find the function $\mathcal{F}(\cdot) = \mathcal{F}(f;n;x,\cdot)$ such that $E_n(\mathcal{F}^{\psi}_{\beta})_{L_1} = E_n(f^{\psi}_{\beta})_{L_1}$ and the following equality takes place

$$|\tilde{\rho}_n(\mathcal{F};x)| = \frac{2}{\pi}\left|\sin\frac{2n-1}{2}x\right| \psi(n)\lambda(n)\left(1 + \xi_1\alpha(n) + \frac{\xi_2}{\lambda(n)}\right) E_n(f^{\psi}_{\beta})_{L_1},$$

where $-4(1+2\pi) \leq \xi_1 < \frac{8}{3}(1+\pi)$, $-(1+2\pi) \leq \xi_2 \leq 2(1+\pi)$.

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[2] O. I. Stepanets', A.S. Serdyuk, A.L. Shydlich, *On some new criteria for infinite differentiability of periodic functions*, Ukr. Math. J., 59, No. 10, 1399-1409 (2007).

Fundamental solution of non-Archimedean pseudo-differential equation of p-adic argument

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The Vladimirov-Taibleson pseudo-differential operator D^α plays a role of a differential operator in the p-adic analysis (see [1, 3]). The analogue of the wave equation for radial functions in t on non-Archimedean spaces

$$D_t^\alpha u - D_x^{\alpha,n} u = 0 \tag{1}$$

was studied in [2].

In present work the fundamental solution of a more general Cauchy problem for the functions of two p-adic variables, radial in t, was found. The main result is stated in Theorem 2. Theorem 3 proves the uniqueness of the solution in the Lizorkin space of locally constant functions $\Phi(\mathbb{Q}_p^n)$, and Theorem 4 gives an estimate of the norm of the solution in L^1-space.

Let $0 < \alpha < 1$, $\beta > 0$. Consider the eigenvalue problem

$$D^\alpha u = \lambda u,\ \lambda = p^{\beta N},\ N \in \mathbb{Z}, \tag{2}$$

where u is not identically zero.

We also suppose that

$$\beta = K\alpha \text{ for some } K \in \mathbb{N}. \tag{3}$$

Proposition 1. *If the condition (3) holds, the equation (2) has the set of solutions in $\Phi(\mathbb{Q}_p)$ of the following form for $N \in \mathbb{Z}$:*

$$u_N(t) = \begin{cases} C_N p^{KN}(1 - \frac{1}{p}), & |t|_p \le p^{-KN}; \\ -C_N p^{KN-1}, & |t|_p = p^{-KN+1}; \\ 0, & |t|_p \ge p^{-KN+2}. \end{cases} \tag{4}$$

Let $0 < \alpha < 1$, $\beta > 0$. We consider the Cauchy problem

$$D_{|t|_p}^\alpha u(|t|_p, x) - D_x^\beta u(|t|_p, x) = 0,\ (t,x) \in \mathbb{Q}_p^+ \times \mathbb{Q}_p^n, \tag{5}$$

$$u(0,x) = u_0(x),\ x \in \mathbb{Q}_p^n, \tag{6}$$

where $u = u(|t|_p, x)$ is a radial function with respect to t, $n \ge 1$.

Theorem 2. *Let $0 < \alpha < 1$, $\beta > 0$ such that the condition (3) holds. Suppose that the function u_0 is in $\Phi(\mathbb{Q}_p)^n$. Then the Cauchy problem (5)-(6) has a solution $u = u(|t|_p, x)$, radial in t, that belongs to the space $\Phi(\mathbb{Q}_p^+)$ for each $x \in \mathbb{Q}_p$, and belongs to $\Phi(\mathbb{Q}_p^n)$ for each $t \in \mathbb{Q}_p^+$.*

If the condition (3) does not hold, then the equation (5) has only a zero solution $u(t,x) \equiv 0$, $t, x \in \mathbb{Q}_p^+ \times \mathbb{Q}_p$.