International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of

Dvytro Grave

(25.08.1863 - 19.12.1939)

Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

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Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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Thurston norm and Euler classes of bounded mean curvature foliations on hyperbolic 3-Manifolds

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Let M be a closed, oriented 3-manifold, and suppose that M contains no non-separating 2 - spheres or tori. For example, M is a closed oriented hyperbolic 3-Manifold.

The Thurston norm on $H_2(M,\mathbb{Z})$ is defined as follows ([1]):

$$||a||_{Th} = inf\{\chi_-(\Sigma)|\ \Sigma \text{ is an embedded oriented surface representing } a \in H_2(M,\mathbb{Z})\}, \quad (1)$$

where $\chi_-(\Sigma) = max\{-\chi(\Sigma), 0\}$. Recall that $\chi(\Sigma) = 2-2g$ denotes the Euler characteristic of a surface Σ of genus g. When Σ is not connected, define $\chi_-(\Sigma)$ to be the sum $\chi_-(\Sigma_1)+\cdots+\chi_-(\Sigma_k)$, where $\Sigma_i,\ i=1,\ldots,k$ are the connected components of Σ. As Thurston showed, the Thurston norm can be extended in a unique way to the norm in $H_2(M,\mathbb{R})$.

The dual Thurston norm can be defined on $H^2(M,\mathbb{R})$ by the formula

$$||\alpha||^*_{Th} = \sup_{\Sigma} \frac{<\alpha,[\Sigma]>}{2g(\Sigma)-2}, \quad (2)$$

where $\alpha \in H^2(M,\mathbb{R})$ and the supremum being taken over all connected, oriented surfaces Σ embedded in M whose genus g is at least 2.

Recall that a *taut* foliation is a codimension one foliation of a closed manifold with the property that every leaf meets a transverse circle. Equivalently, by a result of Dennis Sullivan [2], a codimension one foliation is taut if there exists a Riemannian metric that makes each leaf a minimal surface. Thurston proved that the convex hull of the Euler classes of taut foliations on M is the unit ball for the dual Thurston norm. In particular, the Thurston norm $||e(\mathcal{F})||^*_{Th}$ of the Euler class $e(\mathcal{F}) \in H^2(M,\mathbb{R})$ of a taut foliation $\mathcal{F}$ is no more then one.

We represent the following result.

Theorem 1. *Let M be a closed oriented hyperbolic 3-Manifold and $\mathcal{F}$ be a two-dimensional transversely oriented foliation $\mathcal{F}$ whose leaves have the modulus of mean curvature bounded above by the fixed positive constant H_0. Then*

– *If $H_0 \leq 1$, we have $\mathcal{F}$ is taut and $||e(\mathcal{F})||^*_{Th} = 1$.*

– *If $H_0 > 1$, we have*

$$||e(\mathcal{F})||^*_{Th} \leq 2\pi \frac{1600 H_0^2 Vol(M)^2}{C_0^3 inj(M)} + \frac{300 Vol(M)}{inj(M)} + 1,$$

where $C_0 = 2\min\{inj(M), (\coth)^{-1}(H_0)\}$*,* $Vol(M)$ *is the volume of* M *and* $inj(M)$ *is the injectivity radius of* M*.*

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Nijenhuis geometry and its applications

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This presentation is focused on some results of the long-term research programme *Nijenhuis Geometry* initiated several years ago in cooperation with Vladimir Matveev and Andrey Konyaev.

A *Nijenhuis operator* $L = \big(L^i_j(x)\big)$ is defined to be a field of endomorphisms on a smooth manifold M such that its Nijenhuis torsion identically vanishes, i.e.,

$$\mathcal{N}_L(\xi, w) = L^2[\xi, \eta] + [L\xi, L\eta] - L[L\xi, \eta] - L[\xi, L\eta] = 0, \tag{1}$$

for arbitrary vector fields ξ, η on M. The pair (M, L) is called a *Nijenhuis manifold*.

Relation (1) is the simplest differential-geometric condition on a field of endomorphisms, and that is the reason why Nijenhuis operators appear in many areas of differential geometry and mathematical physics. In the theory of integrable bi-Hamiltonian systems, they serve as recursion operators and their role in this area has been well understood for many years due to pioneering works by F. Magri, Y. Kosmann-Schwarzbach and F. Turiel. A classical fact in complex geometry is that an almost complex structure is integrable if and only if it is Nijenhuis (Newlander–Nireberg theorem). In the context of metric projective geometry, Nijenhuis operators played a crucial role in various classification problems (AB and V. Matveev). They naturally occur in the study of infinite dimensional Poisson brackets of hydrodynamic type (E. Ferapontov *et al*). Even in algebra, Nijenhuis operators turns out to be useful in the theory of integrable systems on Lie algebras and Lie pencils (A. Panasyuk), and also appear as left symmetric algebras.

Besides various applications, our motivation is as follows. Classical geometries are defined by means of a tensor of order 2. For Riemannian, sub-Riemannian, symplectic and Poisson structures, this tensor is a bilinear form (co- or contravariant, symmetric or skew-symmetric). In this list, one type of tensors is still missing: linear operators. Nijenhuis geometry would be a very natural candidate to fill this gap.

Thus, *Nijenhuis Geometry* research programme is aimed at systematic development of the theory of Nijenhuis manifolds. Our vision and first results are presented in [1–8]. More specifically, our goal is to *re-direct the research agenda* in this area from *tensor analysis at generic points* to studying *singularities and global properties*. The ultimate goal of our research programme is to answer three fundamental questions:

(A) **Local description:** to what form can one bring a Nijenhuis operator near almost every point by a local coordinate change?

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