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Online Conference



**Algebraic
and Geometric
Methods of Analysis**

dedicate to the memory
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LIST OF TOPICS

- Topological methods in analysis
- Geometric problems of complex and mathematical analysis
- Algebraic methods in geometry
- Differential geometry in the whole
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Geometric and topological methods in natural sciences

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Painlevé VI Solutions From Equivariant ADHM Instanton Bundles

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We report on the paper [4]. Hitchin [3] had produced a pair of solutions λ_0^\pm for the Painlevé VI differential equation from an $SL_2(\mathbb{C})$ action on the trivial bundle $E_0 \rightarrow P^3$ over complex projective space. We generalize to produce PVI solutions λ_m^\pm for each nonnegative integer m from $SL_2(\mathbb{C})$ actions on the equivariant instanton bundles $E_m \rightarrow P^3$ constructed in [2] via an equivariant version of the Atiyah-Drinfeld-Hitchin-Manin construction [1].

Theorem 1. *For each nonnegative integer m , the equivariant instanton bundle E_m yields a pair of explicitly computable algebraic Painlevé VI solutions $\lambda_m^\pm(t)$, expressed implicitly in terms of the rational function*

$$t(w) = \frac{(1+w)(-3+w)^3}{(-1+w)(3+w)^3}$$

and a rational function of the form

$$\lambda_m^\pm(w) = \left(\frac{(-3+w)^2}{(-1+w)(3+w)} \right) \frac{(-1+w^2)f_m^\pm(w) + 8g_m^\pm(w)}{(3+w^2)f_m^\pm(w) - 24g_m^\pm(w)},$$

where f_m^\pm and g_m^\pm are even polynomials of degree at most $2m(m+1)$.

We have found explicit Okamoto transformations $Q^{\pm 1}$ relating the two hierarchies of solutions λ_m^\pm in a manner reminiscent of the familiar *creation operators* for eigenstates of the quantum harmonic oscillator. The following was proved case-by-case for a finite number of nonnegative integers m , and conjectured to hold for all nonnegative integers m :

Theorem 2. *For each nonnegative integer $m \leq 4$,*

$$\lambda_m^+ = Q^m \lambda_0^+, \quad \lambda_m^- = Q^{-m} \lambda_0^-.$$

We interpret each “creation operator” $Q^{\pm 1}$ as a “shadow” of a putative creation operator for equivariant instanton bundles E_m , which is indicated by the dashed arrows in the summary diagram:

$$\begin{array}{ccccccccc} \lambda_0^+ & \xrightarrow{Q} & \lambda_1^+ & \xrightarrow{Q} & \lambda_2^+ & \xrightarrow{Q} & \lambda_3^+ & \xrightarrow{Q} & \lambda_4^+ \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ E_0 & \dashrightarrow & E_1 & \dashrightarrow & E_2 & \dashrightarrow & E_3 & \dashrightarrow & E_4 \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ \lambda_0^- & \xrightarrow{Q^{-1}} & \lambda_1^- & \xrightarrow{Q^{-1}} & \lambda_2^- & \xrightarrow{Q^{-1}} & \lambda_3^- & \xrightarrow{Q^{-1}} & \lambda_4^- \end{array}$$

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