



International  
Scientific Conference



# Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of  
**Dvytro Grave**  
(25.08.1863 - 19.12.1939)  
Academician of the Ukrainian  
Academy of Sciences, the  
first director of the Institute of  
Mathematics of NAS of Ukraine

May 29 – June 1, 2023  
Odesa, Ukraine

## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Dynamics in nilpotent groups and deformations of locally symmetric rank one manifolds

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We create some analogue of the Sierpiński carpet for nilpotent geometry on horospheres in symmetric rank one negatively curved spaces  $H_{\mathbb{F}}^n$  over division algebras  $\mathbb{F} \neq \mathbb{R}$ , i.e over complex  $\mathbb{C}$ , quaternionic  $\mathbb{H}$ , or octonionic/Cayley numbers  $\mathbb{O}$ . The original Sierpiński carpet in the plane was described by Waclaw Sierpiński in 1916 as a fractal generalizing the Cantor set.

Deforming such a Sierpiński carpet with a positive Lebesgue measure at the sphere at infinity  $\partial H_{\mathbb{F}}^n$  by its "stretching" compatible with nilpotent geometry, we construct a non-rigid discrete  $\mathbb{F}$ -hyperbolic groups  $G \subset \text{Isom } H_{\mathbb{F}}^n$  whose limit set  $\Lambda(G)$  is the whole sphere at infinity  $\partial H_{\mathbb{F}}^n$ . This answers questions by G.D. Mostow [6], L. Bers [4] and S.L. Krushkal [5] about uniqueness of a conformal or CR structure on the sphere at infinity  $\partial H_{\mathbb{F}}^n$  compatible with the action of a discrete isometry group  $G \subset \text{Isom } H_{\mathbb{F}}^n$ .

Previously, for the real hyperbolic spaces, this problem was solved by Apanasov [1, 2]. Due to D. Sullivan [7] rigidity theorem generalized by Apanasov [2] and [3], Theorem 5.19, the complement of the constructed class of discrete groups  $G \subset \text{Isom } H_{\mathbb{F}}^n$  (having a positive Lebesgue measure of the set of vertices of its fundamental polyhedra at infinity) whose limit set  $\Lambda(G)$  is the whole sphere at infinity  $\partial H_{\mathbb{F}}^n$  consists of groups rigid in the sense of Mostow.

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