International
Scientific Conference

Algebraic and Geometric Methods of Analysis

Devoted to 160 anniversary of
Dvytro Grave
(25.08.1863 - 19.12.1939)
Academician of the Ukrainian Academy of Sciences, the first director of the Institute of Mathematics of NAS of Ukraine

May 29 – June 1, 2023
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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HN: semi-boundary saddle node (node) - two;
HS: semi-boundary saddle node (saddle) – four;
BDN: double nod on the boundary – two;
BDNH: double node with a homoclinic boundary – two;
HSC: semi-boundary saddle connection – two;
BSC: a connection of saddles on the boundary – three.
If the boundary is a parabolic cycle:
BPC: boundary parabolic cycle – two flow structures.

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Convex bodies of constant width with exponential illumination number

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Borsuk's number $f(n)$ is the smallest integer such that any set of diameter 1 in the n-dimensional Euclidean space can be covered by $f(n)$ sets of smaller diameter. Currently best known asymptotic upper bound $f(n) \leq (\sqrt{3/2} + o(1))^n$ was obtained by Shramm (1988) and by Bourgain and Lindenstrauss (1989) using different approaches. Bourgain and Lindenstrauss estimated the minimal number $g(n)$ of open balls of diameter 1 needed to cover a set of diameter 1 and showed $1.0645^n \leq g(n) \leq (\sqrt{3/2} + o(1))^n$. On the other hand, Schramm used the connection $f(n) \leq h(n)$, where $h(n)$ is the illumination number of n-dimensional convex bodies of constant width, and showed $h(n) \leq (\sqrt{3/2} + o(1))^n$. The best known asymptotic lower bound on $h(n)$ is subexponential and is the same as for $f(n)$, namely $h(n) \geq f(n) \geq 1.2255^{\sqrt{n}}$ for large n established by Raigorodskii (1999). In 2015 Kalai asked if an exponential lower bound on $h(n)$ can be proved.

We show $h(n) \geq (\cos(\pi/14) + o(1))^{-n}$ by constructing the corresponding n-dimensional bodies of constant width, which answers Kalai's question in the affirmative. The construction is based on a geometric argument combined with a probabilistic lemma establishing the existence of a suitable covering of the unit sphere by equal spherical caps having sufficiently separated centers. The lemma also allows to improve the lower bound of Bourgain and Lindenstrauss to $g(n) \geq (2/\sqrt{3} + o(1))^n \approx 1.1547^n$.

Bifurcation points in random dynamical systems

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Let (M, ρ) be a locally compact separable metric space. By a continuous flows of mappings of M we will understand a family $(\theta_{s,t})_{-\infty<s\leq t<\infty}$, such that

- for all $s \leq t$ $\theta_{s,t} : M \to M$;
- for all $(s, x) \in \mathbb{R} \times M$ the mapping $t \mapsto \theta_{s,t}(x)$ is continuous and satisfies $\theta_{s,s}(x) = x$;
- for all $r \leq s \leq t$ $\theta_{s,t} \circ \theta_{r,s} = \theta_{r,t}$.

If $(\theta_{s,t})_{-\infty<s\leq t<\infty}$ is a continuous flow of mappings of M and $\mathcal{D} = \{(s_n, x_n) : n \geq 1\}$ is a countable dense set in $\mathbb{R} \times M$, then one can consider a sequence of continuous functions $\Phi_n(t) = \theta_{s_n,t}(x_n)$, $t \in [s_n, \infty)$, with the property

$$\max(s_n, s_m) \leq s, \Phi_n(s) = \Phi_m(s) \Rightarrow \Phi_n|_{[s,\infty)} = \Phi_m|_{[s,\infty)} \qquad (1).$$

We are interested in the problem of recovering the flow $(\theta_{s,t})_{-\infty<s\leq t<\infty}$ from the sequence of continuous functions $(\Phi_n)_{n\geq 1}$, $\Phi_n \in C([s_n, \infty), M)$, that satisfy (1). Such problem naturally arises in the theory of stochastic flows. For example, if $\theta_{s,\cdot}(x)$ denotes the solution of the stochastic differential equation

$$dX(t) = a(X(t))dt + b(X(t))dW(t), \ X(s) = x, \qquad (2)$$

where W is a Brownian motion and a and b are continuously differentiable functions bounded together with their derivatives, then for all $r \leq s \leq t$ and $x \in M$, $\theta_{s,t}(\theta_{r,s}(x)) = \theta_{r,t}(x)$ almost surely. However, the equality $\theta_{s,t} \circ \theta_{r,s} = \theta_{r,t}$ may not hold simultaneously for all $r \leq s \leq t$. This fact limits the possibility to apply the dynamic systems technique to the study of stochastic flows. The usual way to deal with this issue is to consider solutions of (2) for some dense sequence of initial values (s_n, x_n) and define solutions for other initial values by a limiting procedure. This strategy works well for stochastic flows of solutions to stochastic differential equations with sufficiently regular coefficients [1]. However, a lot of important stochastic flows are either generated by singular stochastic differential equations, or are not generated by stochastic differential equations at all [2]. This motivates the general question of a possibility to extend a sequence of continuous mappings $(\Phi_n)_{n\geq 1}$ that satisfies