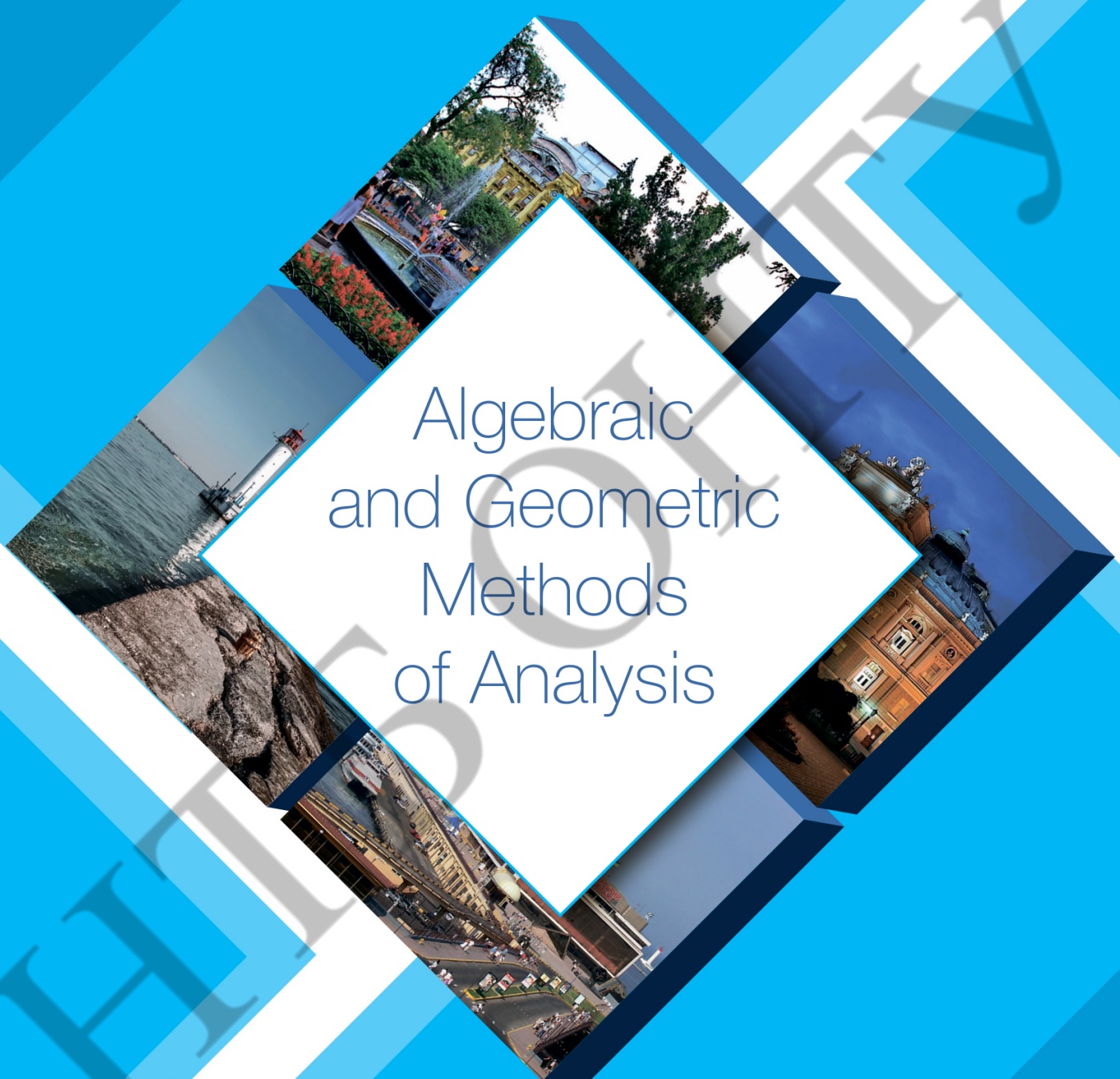


International  
Scientific Conference



Algebraic  
and Geometric  
Methods  
of Analysis

27-30 May 2024  
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

#### LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Group action on noncommutative curves

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Recall that a *noncommutative curve* (noc) is a pair  $\mathbb{X} = (X, \mathcal{O}_{\mathbb{X}})$ , where  $X$  is an algebraic curve (the *base curve* of  $\mathbb{X}$ ) over a field  $\mathbb{k}$  and  $\mathcal{O}_{\mathbb{X}}$  is a sheaf of  $\mathcal{O}_X$ -algebras coherent as a sheaf of  $\mathcal{O}_X$ -modules. We always suppose that  $\mathcal{O}_X \subseteq \mathcal{O}_{\mathbb{X}}$  and the curve  $\mathbb{X}$  is *reduced*, i.e.  $\mathcal{O}_{\mathbb{X}}$  has no nilpotent ideals. We also suppose that  $\mathbb{k}$  is algebraically closed.

The noc  $\mathbb{X}$  is called *hereditary* if  $\text{gl.dim } \mathcal{O}_{\mathbb{X}} = 1$ , that is  $\mathcal{E}xt_{\mathcal{O}_{\mathbb{X}}}^n(\mathcal{M}, \mathcal{N}) = 0$  for  $n > 1$  and all coherent sheaves of  $\mathcal{O}_{\mathbb{X}}$ -modules  $\mathcal{M}, \mathcal{N}$ . We denote by  $\mathbb{X}^\sharp$  the noc  $(\mathcal{O}_X, \mathcal{O}_{\mathbb{X}^\sharp})$ , where for every point  $x \in X$

$$\mathcal{O}_{\mathbb{X}^\sharp, x} = \begin{cases} \mathcal{O}_{\mathbb{X}, x} & \text{if } \mathcal{O}_{\mathbb{X}, x} \text{ is hereditary,} \\ \mathcal{E}nd_{\mathcal{O}_{\mathbb{X}, x}} \mathfrak{r}_x & \text{otherwise,} \end{cases}$$

where  $\mathfrak{r}_x = \text{rad } \mathcal{O}_{\mathbb{X}, x}$ . It is known that if  $\mathfrak{r}_x = \text{rad } \mathcal{O}_{\mathbb{X}^\sharp, x}$ , the noc  $\mathbb{X}$  is hereditary. In this case  $\mathbb{X}$  is called a *Bäckström curve*. If, moreover,  $\ell_{\mathcal{O}_{\mathbb{X}^\sharp}}(\mathcal{O}_{\mathbb{X}^\sharp} \otimes_{\mathcal{O}_{\mathbb{X}}} U) \leq 2$  for every simple sheaf of  $\mathcal{O}_{\mathbb{X}}$ -modules  $U$ ,  $\mathbb{X}$  is called a *nodal curve*. Note that a “usual” (commutative) curve  $X$  is nodal if and only if all its singularities are *simple nodes*, that is, if  $x \in X$  is a singular point,  $\hat{\mathcal{O}}_x \simeq \mathbb{k}[[x, y]]/(xy)$ . The structure of nodal nocs is described in [1].

Let a finite group  $G$  acts on a noc  $\mathbb{X}$ . It means that it acts on the base curve  $X$  as well as on the sheaf of algebras  $\mathcal{O}_{\mathbb{X}}$  (maybe with a factor set in the sense of [2]). The noc  $\mathbb{X} * G = (X/G, \mathcal{O}_{\mathbb{X} * G})$  (the *crossed product* of  $\mathbb{X}$  and  $G$ ) is defined. Note that  $G$  naturally acts on the category  $\text{Coh } \mathbb{X}$  of coherent  $\mathcal{O}_{\mathbb{X}}$ -modules.

**Theorem 1.** *Suppose that the order of the group  $G$  is invertible in  $\mathbb{k}$ .*

- (1) *There is an equivalence of categories  $\text{Coh}(\mathbb{X} * G)$  and  $\text{add}((\text{Coh } \mathbb{X}) * G)$ , where  $\text{add } \mathcal{C}$  denotes the Karubian closure of the category  $\mathcal{C}$ , i.e. the smallest additive category containing  $\mathcal{C}$  and such that all idempotents in it split.*
- (2) *If  $\mathbb{X}$  is hereditary (Bäckström, nodal), so is  $\mathbb{X} * G$ .*

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## 6D-Riemannian metric associated at the Navier-Stokes equations and its applications

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**Theorem 1.** *The 6D metric in local coordinates  $(t, x, y, z, v, w)$*

$$d_s^2 = -2B(t, x, y, z) d_t d_v + 2E(t, x, y, z) d_t d_w + d_t d_x + 2H(t, x, y, z) d_v d_w + d_v d_y -$$

$$- 2 \int \frac{\partial}{\partial y} H(t, x, y, z) dz d_w^2 + d_w d_z,$$

with conditions on coefficients

$$\begin{aligned} \frac{\partial}{\partial y} H(t, x, y, z) - \frac{\partial}{\partial x} E(t, x, y, z) &= 0, & \frac{\partial}{\partial z} H(t, x, y, z) - \frac{\partial}{\partial x} B(t, x, y, z) &= 0, \\ \frac{\partial}{\partial z} E(t, x, y, z, t) - \frac{\partial}{\partial y} B(t, x, y, z) &= 0 \end{aligned}$$

is associated with equations of compatibility of the Navier-Stokes system of equations

$$\vec{\nabla} P(\vec{t}, x) = \frac{\partial}{\partial t} \vec{V} + (\vec{V} \cdot \vec{\nabla}) \vec{V} = \mu \Delta \vec{V}, \quad \vec{\nabla} \cdot \vec{V} = 0$$

with respect to the function of pressure in flow of liquid  $P(\vec{t}, x) = P(t, x, y, z)$ . [1],[2]

On base of this theorem the examples of exact solutions of Navier-Stokes system of equations may be constructed.

The Ricci-tensor  $R_{ik}$  of given metric has six components  $R_{tt}, R_{vv}, R_{vw}, R_{ww}, R_{tv}, R_{tw}$  and from conditions of compatibility between the various equations for  $R_{ik}$  lead determined the components of velocities  $\vec{U}(t, \vec{x})$  and the function pressure  $P(t, \vec{x})$ .

As example simplest reduction of considered metric has the form

$$\begin{aligned} ds^2 = & -2 \left( \frac{\partial^2}{\partial z^2} K(t, x, y, z) \right) d_t d_v + 2 \left( \frac{\partial^2}{\partial z \partial y} K(t, x, y, z) \right) d_t d_w + d_t d_x + \\ & + 2 \left( \frac{\partial^2}{\partial z \partial x} K(t, x, y, z) \right) d_v d_w + d_v d_y - 2 \left( \frac{\partial^2}{\partial y \partial x} K(t, x, y, z) \right) d_w^2 + d_w d_z. \end{aligned}$$

As example the metric is the Ricci-flat for the flows of the form

$$U(t, x, y, z) = -1/2x \frac{\partial}{\partial z} F(t, z), \quad V(t, x, y, z) = -1/2y \frac{\partial}{\partial z} F(t, z), \quad W(t, x, y, z) = F(t, z),$$

with the function  $F(t, z)$  which is determined from the equation

$$-\mu \frac{\partial^3}{\partial z^3} F(t, z) + F(t, z) \frac{\partial^3}{\partial z^3} F(t, z) + \frac{\partial^2}{\partial z \partial t} F(t, z) = 0,$$

for which

$$\begin{aligned} B(t, \vec{x}) &= \frac{\partial}{\partial t} F(t, z) + F(t, z) \frac{\partial}{\partial z} F(t, z) \\ H(t, \vec{x}) &= -1/2x \frac{\partial^2}{\partial z \partial t} F(t, z) + 1/4x \frac{\partial}{\partial z} F(t, z)^2 - \\ &\quad - 1/2xF(t, z) \frac{\partial^2}{\partial z \partial z} F(t, z) + 1/2\mu x \frac{\partial^3}{\partial z \partial z \partial z} F(t, z), \\ E(t, \vec{x}) &= -1/2y \frac{\partial^2}{\partial z \partial t} F(t, z) + 1/4y \frac{\partial}{\partial z} F(t, z)^2 - \\ &\quad - 1/2yF(t, z) \frac{\partial^2}{\partial z \partial z} F(t, z) + 1/2\mu y \frac{\partial^3}{\partial z \partial z \partial z} F(t, z). \end{aligned}$$

The metric of the form

**Theorem 2.**

$$\begin{aligned}
ds^2 = & 2 dx^2 + 2 dx dy + 2 dx du + 2 dy^2 + 2 dy dz + 2 dy dv + 2 dz^2 + 2 dz dw + \\
& + 2 dt dp + 2 d\eta d\xi + 2 dp d\chi + 2 dm dn + A dt^2 + B d\eta^2 + C d\rho^2 + E dm^2,
\end{aligned} \tag{1}$$

where

$$A = 2 - U(x, y, z, t)u - V(x, y, z, t)v - W(x, y, z, t)w,$$

$$B = \left(-UW + \mu \frac{\partial}{\partial z}U\right)w + \left(-UV + \mu \frac{\partial}{\partial y}U\right)v + \left(\mu \frac{\partial}{\partial x}U - (U)^2 - P\right)u - Up,$$

$$C = \left(-VW + \mu \frac{\partial}{\partial z}V\right)w + \left(\mu \frac{\partial}{\partial y}V - (V)^2 - P\right)v + \left(-UV + \mu \frac{\partial}{\partial x}V\right)u - Vp,$$

$$E = \left(-\mu \frac{\partial}{\partial x}U - \mu \frac{\partial}{\partial y}V - (W)^2 - P\right)w + \left(-VW + \mu \frac{\partial}{\partial y}W\right)v + \left(-UW + \mu \frac{\partial}{\partial x}W\right)u - Wp$$

is the Ricci-flat on solutions of Navier-Stokes system of equations.

Geometric characteristics of the 6D- metric depend on choice of the functions  $H(t, \vec{x})$ ,  $B(t, \vec{x})$ ,  $E(t, \vec{x})$  and considerations of their properties joint with the metric of 14D-space can be used to determination of the functions

$$U(t, \vec{x}), \quad V(t, \vec{x}), \quad W(t, \vec{x}), \quad P(t, \vec{x})$$

from the NS-equations:

$$\begin{aligned}
& \frac{\partial}{\partial t}U(t, \vec{x}) + U(t, \vec{x}) \frac{\partial}{\partial x}U(t, \vec{x}) + V(t, \vec{x}) \frac{\partial}{\partial y}U(t, \vec{x}) + W(t, \vec{x}) \frac{\partial}{\partial z}U(t, \vec{x}) - \\
& - \mu \left( \frac{\partial^2}{\partial x^2}U(t, \vec{x}) + \frac{\partial^2}{\partial y^2}U(t, \vec{x}) + \frac{\partial^2}{\partial z^2}U(t, \vec{x}) \right) + H(t, \vec{x}) = 0,
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial t}V(t, \vec{x}) + U(t, \vec{x}) \frac{\partial}{\partial x}V(t, \vec{x}) + V(t, \vec{x}) \frac{\partial}{\partial y}V(t, \vec{x}) + W(t, \vec{x}) \frac{\partial}{\partial z}V(t, \vec{x}) - \\
& - \mu \left( \frac{\partial^2}{\partial x^2}V(t, \vec{x}) + \frac{\partial^2}{\partial y^2}V(t, \vec{x}) + \frac{\partial^2}{\partial z^2}V(t, \vec{x}) \right) + E(t, \vec{x}) = 0,
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial}{\partial t}W(t, \vec{x}) + U(t, \vec{x}) \frac{\partial}{\partial x}W(t, \vec{x}) + V(t, \vec{x}) \frac{\partial}{\partial y}W(t, \vec{x}) + W(t, \vec{x}) \frac{\partial}{\partial z}W(t, \vec{x}) - \\
& - \mu \left( \frac{\partial^2}{\partial x^2}W(t, \vec{x}) + \frac{\partial^2}{\partial y^2}W(t, \vec{x}) + \frac{\partial^2}{\partial z^2}W(t, \vec{x}) \right) + B(t, \vec{x}) = 0
\end{aligned}$$

$$\frac{\partial}{\partial x}U(t, \vec{x}) + \frac{\partial}{\partial y}V(t, \vec{x}) + \frac{\partial}{\partial z}W(t, \vec{x}) = 0.$$

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