International
Scientific Conference

Algebraic and Geometric Methods of Analysis

26-30 may 2020
Odesa, Ukraine

List of topics

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

Organizers

- Ministry of Education and Science of Ukraine
- Odesa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Odessa I. I. Mechnikov National University
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

Program Committee

Chairman: Prishlyak A.
(*Kyiv, Ukraine*)
Balan V.
(*Bucharest, Romania*)
Banakh T.
(*Lviv, Ukraine*)
Bolotov D.
(*Kharkiv, Ukraine*)
Borysenko O.
(*Kharkiv, Ukraine*)
Cherevko Ye.
(*Odesa, Ukraine*)
Fedchenko Yu.
(*Odesa, Ukraine*)
Karlova O.
(*Chernivtsi, Ukraine*)
Kiosak V.
(*Odessa, Ukraine*)
Kirillov V.
(*Odesa, Ukraine*)
Konovenko N.
(*Odesa, Ukraine*)
Lyubashenko V.
(*Kyiv, Ukraine*)
Maksymenko S.
(*Kyiv, Ukraine*)
Matsumoto K.
(*Yamagata, Japan*)
Mormul P.
(*Warsaw, Poland*)
Mykhailyuik V.
(*Chernivtsi, Ukraine*)
Plachta L.
(*Krakov, Poland*)
Pokas S.
(*Odesa, Ukraine*)
Polulyakh E.
(*Kyiv, Ukraine*)
Sabitov I.
(*Moscow, Russia*)
Savchenko A.
(*Kherson, Ukraine*)
Sergeeva A.
(*Odesa, Ukraine*)
Shelekhov A.
(*Tver, Russia*)
Volkov V.
(*Odesa, Ukraine*)
Zarichnyi M.
(*Lviv, Ukraine*)

Administrative committee

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Svytyy I., Dean of the Faculty of Computer Systems and Automation.

Organizing commitee

Kirillov V.	Maksymenko S.	Osadchuk E.
Konovenko N.	Cherevko Ye.	Prus A.
Fedchenko Yu.		

Minimal generating set and structure of a wreath product of groups and the fundamental group of an orbit of Morse function

Skuratovskii Ruslan
(The National Technical University of Ukraine Igor Sikorsky Kiev Polytechnic Institute, Ukraine)
E-mail: r.skuratovskii@kpi.ua

Aled Williams
(Cardiff University, Cardiff, UK)
E-mail: williamsae13@cardiff.ac.uk

The quotient group of the restricted and unrestricted wreath product by its commutator is found. The generic sets of commutator of wreath product were investigated.

We generalize the results presented in the book of Meldrum J. [1] about commutator subgroup of wreath products since, as well as considering regular wreath products, we consider those which are not regular (in the sense that the active group $\mathcal{A}$ does not have to act faithfully). The fundamental group of orbits of a Morse function $f : M \to \mathbb{R}$ defined upon a Möbius band M with respect to the right action of the group of diffeomorphisms $\mathcal{D}(M)$ has been investigated.

Denote the set of all the orbits of $\mathcal{A}$ on X by $\mathcal{O}$, if this set is finite then by $\mathcal{O}_f$. Recall that the direct product indexed by infinite set consists of all infinite sequences, and the direct sum consists only of sequences with finitely many elements distinct from zero. Denote by $Z(\tilde{\triangle}(\mathcal{B}))$ the subgroup of diagonal subgroup [2] $Fun(X, Z(B))$ of functions $f : X \to Z(B)$ which are constant on each orbit of action of A on X for unrestricted wreath product, and denote by $Z(\triangle(\mathcal{B}^n))$ the subgroup of diagonal $Fun(X, Z(\mathcal{B}^n))$ of functions with the same property for restricted wreath product, where n is number of non-trivial coordinates in base of wreath product.

Theorem 1. *A centre of the group $(\mathcal{A}, X) \wr \mathcal{B}$ is direct product of normal closure of centre of a diagonal of $Z(\mathcal{B}^n)$ i.e. $(E \times Z(\triangle(\mathcal{B}^n)))$, trivial an element, and intersection of $(\mathcal{K}) \times E$ with $Z(\mathcal{A})$. In other words,*

$$Z((\mathcal{A}, X) \wr \mathcal{B}) = \langle (1;\ \underbrace{h, h, \ldots, h}_{n}),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\triangle(\mathcal{B}^n)),$$

where $h \in Z(\mathcal{B})$, $|X| = n$.

For restricted wreath product with n non-trivial coordinate: $Z((\mathcal{A}, X) \wr \mathcal{B}) =$
$\langle (1;\ \ldots, h, h, \ldots, h, \ldots),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\triangle(\mathcal{B}^n)) \simeq$
$\simeq \bigoplus_{j \in O_f} (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\mathcal{B})$.

In case of unrestricted wreath product we have: $Z((\mathcal{A}, X) \wr \mathcal{B}) =$
$\langle (1;\ \ldots, h_{-1}, h_0, h_1, \ldots, h_i, h_{i+1}, \ldots,),\ e,\ Z(\mathcal{K}, X) \wr \mathcal{E} \rangle \simeq (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\tilde{\triangle}(\mathcal{B}))$
$= \prod_{j \in O} (Z(\mathcal{A}) \cap \mathcal{K}) \times Z(\mathcal{B})$.

Theorem 2. *If $W = (\mathcal{A}, X) \wr (\mathcal{B}, Y)$, where $|X| = n$, $|Y| = m$ and active group $\mathcal{A}$ acts on X transitively, then*

$$d\left(G'\right) \leq (n-1)d(\mathcal{B}) + d(\mathcal{B}') + d(\mathcal{A}').$$

Theorem 3. *The quotient group of a restricted wreath products $G = Z \wr_X Z$ by a commutator subgroup is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. In previous conditions if $G = A \wr_X B$ then, $G/G' = A/A' \times B/B'$. If $G = Z_n \wr Z_m$, where $(m,\ n) = 1$, then $d(G/G') = 1$. If $G = Z \wr Z$ is an unrestricted regular wreath product then $G/G' \simeq Z \times E \simeq Z$.*

References

[1] John DP Meldrum. Wreath products of groups and semigroups, volume 74. CRC Press, London, 1995.

[2] *J.D. Dixon, B. Mortimer*. Permutation Groups. Springer-Verlag, New York 1996.

[3] Skuratovskii R. V. The commutator subgroup of Sylow 2-subgroups of alternating group, commutator width of wreath product [Source: Arxiv.org], access mode: https://arxiv.org/pdf/1903.08765.pdf.

[4] Ruslan Skuratovskii. Minimal generating sets for wreath products of cyclic groups, groups of automorphisms of ribe graph and fundumental groups of some morse functions orbits. In *Algebra, Topology and Analysis*, Odessa, Ukraine, 2016, 121–123.

[5] Skuratovskii R. V., Aled Williams, Minimal Generating Set and a Structure of the Wreath Product of Groups, and the Fundamental Group of the Orbit Morse Function. [Source: Arxiv.org], 3 Sep 2019, access mode: https://arxiv.org/pdf/1901.00061.pdf

[6] Skuratovskii R. V. Involutive irreducible generating sets and structure of sylow 2-subgroups of alternating groups. // *ROMAI J.* Vol 13, № 1 (13): 117–139, 2017.