

International
Scientific Conference



Algebraic
and Geometric
Methods
of Analysis

27-30 May 2024
Odesa, Ukraine

The purpose of this conference is to bring together researchers in geometry, topology, algebra, analysis and dynamical systems and to provide for them a forum to present their recent work to colleagues from different nationalities. This way we aim to stimulate discussion about the latest findings in geometrical and topological methods in analysis and to increase international collaboration.

The conference continues the traditional annual conference «Geometry in Odesa» holding from 2004, and hosted by Odesa National University of Technology (Odesa National Academy of Food Technologies till 2021). From 2017 the conference was renamed to «Algebraic and geometric methods of analysis» (AGMA).

The Conference languages: Ukrainian and English.

LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odesa National University of Technology, Ukraine
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Taras Shevchenko National University of Kyiv
- Kyiv Mathematical Society

SCIENTIFIC COMMITTEE

- | | |
|---|--|
| • Vladimir Balan (<i>Bucharest, Romania</i>) | • Volodymyr Lyubashenko (<i>Kyiv, Ukraine</i>) |
| • Taras Banakh (<i>Lviv, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Dmytro Bolotov (<i>Kharkiv, Ukraine</i>) | • Koji Matsumoto (<i>Yamagata, Japan</i>) |
| • Vyacheslav Boyko (<i>Kyiv, Ukraine</i>) | • Piotr Mormul (<i>Warsaw, Poland</i>) |
| • Yulia Fedchenko (<i>Odesa, Ukraine</i>) | • Maryna Nesterenko (<i>Kyiv, Ukraine</i>) |
| • Oleg Gutik (<i>Lviv, Ukraine</i>) | • Roman Popovych (<i>Kyiv, Ukraine</i>) |
| • Olena Karlova (<i>Chernivtsi, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |
| • Volodymyr Kiosak (<i>Odesa, Ukraine</i>) | • Aleksandr Savchenko (<i>Kherson, Ukraine</i>) |
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | |

ORGANIZING COMMITTEE

- | | |
|---|--|
| • Nadiia Konovenko (<i>Odesa, Ukraine</i>) | • Bohdan Mazhar (<i>Kyiv, Ukraine</i>) |
| • Yuliya Fedchenko (<i>Odesa, Ukraine</i>) | • Sergiy Maksymenko (<i>Kyiv, Ukraine</i>) |
| • Mykola Lysynskiy (<i>Kyiv, Ukraine</i>) | • Alexandr Prishlyak (<i>Kyiv, Ukraine</i>) |

Uniform approximation by Fourier sums in Weyl–Nagy classes $W_{\beta,1}^r$

Anatolii Serdyuk

(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)

E-mail: serdyuk@imath.kiev.ua

Ihor Sokolenko

(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)

E-mail: sokol@imath.kiev.ua

We investigate asymptotic behavior of the least upper bounds of approximations in the uniform metric by Fourier sums $S_{n-1}(f; \cdot)$ of classes $W_{\beta,1}^r$ of 2π -periodic Weyl–Nagy differentiable functions f .

Let L_p , $1 \leq p \leq \infty$, and C be the spaces of 2π -periodic functions with standard norms $\|\cdot\|_{L_p}$ and $\|\cdot\|_C$, respectively.

Further, let $W_{\beta,p}^r$, $r > 0$, $\beta \in \mathbb{R}$, $1 \leq p \leq \infty$, be classes of 2π -periodic functions f that can be represented in the form of convolution

$$f(x) = \frac{a_0}{2} + \frac{1}{\pi} \int_{-\pi}^{\pi} \varphi(x-t) B_{r,\beta}(t) dt, \quad a_0 \in \mathbb{R}, \quad (1)$$

with Weyl–Nagy kernels of the form $B_{r,\beta}(t) = \sum_{k=1}^{\infty} k^{-r} \cos(kt - \frac{\beta\pi}{2})$, of function φ satisfying the condition $\varphi \in B_p^0 = \{\varphi \in L_p : \|\varphi\|_p \leq 1, \int_{-\pi}^{\pi} \varphi(t) dt = 0\}$.

The classes $W_{\beta,p}^r$ are called the Weyl–Nagy classes and the function φ in representation (1) is called the (r, β) -derivative of the function f in the Weyl–Nagy sense and denoted by f_{β}^r .

Theorem 1. *Let $r > 2$, $\beta \in \mathbb{R}$, and $n \in \mathbb{N}$. The following estimate is true*

$$\mathcal{E}_n(W_{\beta,1}^r)_C = \sup_{f \in W_{\beta,1}^r} \|f(\cdot) - S_{n-1}(f; \cdot)\|_C = \frac{1}{n^r} \left(\frac{1}{\pi(1 - e^{-r/n})} + \mathcal{O}(1)\delta_{r,n} \right), \quad (2)$$

where $\mathcal{O}(1)$ is a quantity uniformly bounded in all analyzed parameters,

$$\delta_{r,n} = \begin{cases} 1 + \frac{n}{r(r-2)}, & 2 < r \leq n+1, \\ \frac{r}{n^2} e^{-r/n}, & n+1 \leq r \leq n^2, \\ e^{-r/n} & r \geq n^2. \end{cases}$$

Remark 2. Estimate (2) was published for $r \geq \sqrt{n} + 1$ in [1, 2] (2019, 2022).

This work was partially supported by the Grant H2020-MSCA-RISE-2019, project number 873071 (SOMPATY: Spectral Optimization: From Mathematics to Physics and Advanced Technology), by the VolkswagenStiftung project "From Modeling and Analysis to Approximation" and by the grant from the Simons Foundation (1290607, AS) and (1290607, IS)".

REFERENCES

- [1] A. S. Serdyuk and I. V. Sokolenko. Approximation by Fourier sums in classes of differentiable functions with high exponents of smoothness. *Meth. Funct. Anal. Topol.*, 25, No. 4, 381–387, 2019.

[2] A. S. Serdyuk and I. V. Sokolenko. Approximation by Fourier sums in the classes of Weyl–Nagy differentiable functions with high exponents of smoothness. *Ukr. Math. J.*, 74, No 5, 783–800, 2022.

On the asymptotic behavior of solutions to nonlinear Beltrami equation

Igor Petkov

(Admiral Makarov National University of Shipbuilding, Mykolaiv, Ukraine)

E-mail: igorpetkov83@gmail.com

Ruslan Salimov

(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)

E-mail: ruslan.salimov1@gmail.com

Mariia Stefanchuk

(Institute of Mathematics of NAS of Ukraine, Kyiv, Ukraine)

E-mail: stefanmv43@gmail.com

Let \mathbb{C} be the complex plane. In the complex notation $f = u + iv$ and $z = x + iy$, the *Beltrami equation* in a domain $G \subset \mathbb{C}$ has the form

$$f_{\bar{z}} = \mu(z)f_z, \quad (1)$$

where $\mu: G \rightarrow \mathbb{C}$ is a measurable function and

$$f_{\bar{z}} = \frac{1}{2}(f_x + if_y) \quad \text{and} \quad f_z = \frac{1}{2}(f_x - if_y)$$

are formal derivatives of f in \bar{z} and z , while f_x and f_y are partial derivatives of f in the variables x and y , respectively.

We consider the following equation written in the polar coordinates (r, θ)

$$f_{\theta} = \sigma(re^{i\theta})|f_r|^m f_r. \quad (2)$$

We rewrite the equation (2) in the Cartesian form:

$$f_{\bar{z}} = \frac{z}{\bar{z}} \frac{1 + i\sigma(z)|z|^{-m-1}|zf_z + \bar{z}f_{\bar{z}}|^m}{1 - i\sigma(z)|z|^{-m-1}|zf_z + \bar{z}f_{\bar{z}}|^m} f_z. \quad (3)$$

Assuming $m = 0$, the equation (3) also becomes the standard linear Beltrami equation (1) with

$$\mu(z) = \frac{z}{\bar{z}} \frac{1 + i\sigma(z)/|z|}{1 - i\sigma(z)/|z|}.$$

Choosing $m = 0$ and $\sigma = i|z|$ in (3), we arrive at the classical Cauchy-Riemann system. Later on we assume that $m > 0$.

A mapping $f: G \rightarrow \mathbb{C}$ is called *regular at a point* $z_0 \in G$, if f has the total differential at this point and its Jacobian $J_f = |f_z|^2 - |f_{\bar{z}}|^2$ does not vanish. A homeomorphism f of Sobolev class $W_{\text{loc}}^{1,1}$ is called *regular*, if $J_f > 0$ a.e. By a *regular solution of the equation* (3) we call a regular homeomorphism $f: G \rightarrow \mathbb{C}$, which satisfies (3) a.e. in G .

Later on, we use the following notations:

$$B_r = \{z \in \mathbb{C} : |z| < r\}, \quad \mathbb{B} = \{z \in \mathbb{C} : |z| < 1\}$$

and

$$\gamma_r = \{z \in \mathbb{C} : |z| = r\}.$$

V. Oles <i>Computing the Gromov–Hausdorff distance using gradient methods</i>	91
G. Ovando <i>Magnetic trajectories on 2-step nilmanifolds</i>	92
I. Ovtsynov <i>N-foci balls in hyperbolic geometry</i>	93
V. Penhryn, O. Nykyforchyn <i>A retraction from the space of pseudometrics to the space of ultrapseudometrics</i>	94
O. Kozachok, A. Petravchuk <i>Action of derivations on polynomials and on Jacobian derivations</i>	96
E. Petrov <i>Periodic point theorem for mappings contracting total pairwise distance</i>	98
I. Pozdniakova, O. Gutik <i>On the semigroup of non-injective monoid endomorphisms of some extension of the bicyclic monoid</i>	100
A. Prishlyak <i>Structure of gradient bifurcations on compact 2-manifolds</i>	101
V. Prokip <i>About square roots of matrices over factorial domains</i>	102
A. Rasila <i>Results on boundary behavior of quasiregular and harmonic mappings</i>	104
F. A. Rossi <i>Einstein Solvmanifolds not based on Nilsolitons</i>	104
J. Saavedra <i>Ricci flow of G_2-type real flag manifolds</i>	105
A. Serdyuk, T. Stepaniuk <i>Approximation by interpolation trigonometric polynomials on the sets of infinitely differentiable functions</i>	106
M. Serdiuk <i>Fundamental solution of non-Archimedean pseudo-differential equation of p-adic argument</i>	108
M. Serivka, O. Gutik <i>On the semigroup of injective monoid endomorphisms of a some extension of the bicyclic semigroup</i>	109
R. Servadei <i>On some nonlocal critical equations</i>	111
E. Sevost'yanov, O. Dovhopiatyi, N. Ilkevych, M. Androschuk <i>On boundary estimates of mappings, acting onto domains with a locally quasiconformal boundary</i>	111
O. Shugailo <i>Some properties of affine ruled submanifolds</i>	113
H. Sinyukova <i>On some vanishing theorems of global character about geodesic mappings of complete Riemannian spaces</i>	114
R. Skuratovskii <i>Subwreath product as structure of normal subgroups of permutational wreath products</i>	114
A. Serdyuk, I. Sokolenko <i>Uniform approximation by Fourier sums in Weyl–Nagy classes $W_{\beta,1}^r$</i>	117
I. Petkov, R. Salimov, M. Stefanchuk <i>On the asymptotic behavior of solutions to nonlinear Beltrami equation</i>	118