



International  
Scientific Conference



# Algebraic and Geometric Methods of Analysis



Devoted to 160 anniversary of  
**Dvytro Grave**  
(25.08.1863 - 19.12.1939)  
Academician of the Ukrainian  
Academy of Sciences, the  
first director of the Institute of  
Mathematics of NAS of Ukraine

May 29 – June 1, 2023  
Odesa, Ukraine

## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences
- Geometric problems in mathematical analysis

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# Degenerations of complex associative algebras of dimension three

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Let  $\Lambda_3(\mathbb{C}) (= \mathbb{C}^{27})$  be the space of structure vectors of 3-dimensional algebras over  $\mathbb{C}$  considered as a  $G$ -module via the action of  $G = \text{GL}(3, \mathbb{C})$  on  $\Lambda_3(\mathbb{C})$  ‘by change of basis’. We determine the complete degeneration picture inside the algebraic subset  $\mathcal{A}_3^s$  of  $\Lambda_3(\mathbb{C})$  consisting of associative algebra structures via the corresponding information on the algebraic subsets  $\mathcal{L}_3$  and  $\mathcal{J}_3$  of  $\Lambda_3(\mathbb{C})$  of Lie and Jordan algebra structures respectively. This is achieved with the help of certain  $G$ -module endomorphisms  $\phi_1, \phi_2$  of  $\Lambda_3(\mathbb{C})$  which map  $\mathcal{A}_3^s$  onto algebraic subsets of  $\mathcal{L}_3$  and  $\mathcal{J}_3$  respectively.

This is a joint work with Nataliya M. Ivanova

## REFERENCES

- [1] N.M Ivanova and C.A. Pallikaros. Degenerations of complex associative algebras of dimension three via Lie and Jordan algebras. arXiv:2212.10635.

# Several forms of the geometric Lusternik-Schnirel’mann category

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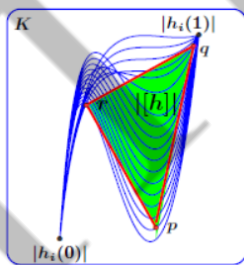


Fig. 1.1 → Fig. 1.2

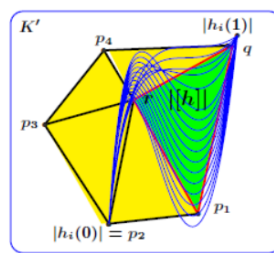


FIGURE 0.1. Fig. 1.1 from Theorem 1.6 & Fig. 1.2 from Corollary 1.8.

This paper introduces results for several forms of the geometric Lusternik-Schirel’mann categories (LS gcat).

## 1. GEOMETRIC LUSTERNIK-SCHNIREL'MANN CATEGORY

Let  $h : I \rightarrow K$  be a continuous map called **homotopy** (briefly, **path** in a space  $K$ ). A **homotopic class** for different maps  $h$  (denoted by  $[h] = \{h_0, \dots, h_i, \dots, h_{n-1}, h_n\}$  with  $[n] = \text{mod } n \in \mathbb{Z}^+$ ) is a collection of  $h_{i[n]}$  homotopic maps that have the same endpoints, namely,  $h_i(0)$  and  $h_i(1)$ . The geometric realization of  $[h]$  (denoted by  $|[h]|$ ) is a collection of sinusoidal curves, each being the geometric realization of a path  $h$ .

**Definition 1.1** (Geometric LS Category). <sup>1</sup> For a topological space  $X$ , the geometric category of  $X$  is the minimal covering of  $X$  with contractible open subsets of  $X$ .

**Lemma 1.2.** *Let  $\mathbf{h}$  be a homotopic sinusoidal path. The geometric realization  $|\mathbf{h}|$  is a planar sinusoidal curve.*

**Lemma 1.3.** *Let  $[\mathbf{h}]$  be a collection of homotopic paths with common endpoints. The geometric realization  $|[h]|$  is a collection space filling planar curves.*

**Lemma 1.4.** *Let  $\Delta pqr$  be a planar filled triangle in a space  $K$ , geometric realization  $|[h]|$  such that each path  $h$  has endpoints  $h(0), h(1) \in K \setminus \Delta pqr$ . Then  $\lim_{i \rightarrow \infty} h_i \in [h] \supseteq \Delta pqr$ .*

**Lemma 1.5.** *Let  $h$  be a homotopic path in space  $K$ .*

- 1° Every path  $\mathbf{h}$  is contractible.
- 2° There exists a minimal  $|[h]|$  in space  $K$  with  $h_i \in [h]$  with the same boundary endpoints  $\mathbf{h}(0), \mathbf{h}(1) \in \partial \Delta pqr$  such that  $|[h]|$  covers triangle  $\Delta pqr \subset K$ .
- 3° Every planar triangle in  $K$  has a minimal covering  $|[h]|$ .

**Theorem 1.6.** *There exists  $gcat(|[h]| \in 2^K)$  such that  $\min |[h]| \supseteq \Delta pqr \in 2^K$ .*

**Example 1.7.** From Theorem 1.6, the triangle  $\Delta pqr$  in Fig. 0.1 has a  $|[h]|$  minimal covering, which is a  $gcat(T)$ .

A **cluster of triangles**  $\{\Delta pqr\}$  in a Euclidean space  $K$  is a collection of triangles attached to a common vertex. From Theorem 1.6, we have

**Corollary 1.8.** *Let  $|\{[h]\}| \subset K$  such that each  $\mathbf{h} \in \{[h]\} \subset K \setminus \{\Delta pqr\}$  has the same endpoints. Then  $\exists gcat(|\{[h]\}|): \min |\{[h]\}| \supseteq \{\Delta pqr\} \in 2^K$ .*

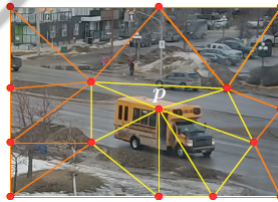


FIGURE 1.2. Delaunay Triangle cluster minimally covering bus in video frame foreground.

<sup>1</sup> L. Montejano, Lusternik schirel'mann category: a geometric approach, Banach Cent. Publ. 18 (1986), 117–129.

**Example 1.9.** From Corollary 1.8, there is a  $gcat(|\{[h]\}|)$  such that  $gcat(|\{[h]\}|)$  is a minimal covering of the triangle cluster  $\{\Delta pqr\} \cap p$ , which is a bounded region in Fig. 0.1.

## 2. MINIMAL VIDEO FRAME FOREGROUND OBJECT COVERING

Delaunay triangulations<sup>2</sup> represent pieces of a continuous space in form of triangles with edges attached to selected vertices<sup>3</sup>.

**Theorem 2.1.** *Let  $\mathcal{TC}$  be a Delaunay triangle cluster with  $k$  triangles minimally covering a planar bounded region  $E \in 2^{\mathbb{R}^2}$ . Then  $gcat(\mathcal{TC}) = k$ .*

**Example 2.2.** With restrictions on the selection of vertices (e.g., centroids), we obtain a minimal cluster  $\mathcal{TC}$  of  $k$  triangles covering a bus, which is a bounded region in a Delaunay triangulation of the video frame foreground in Fig. 1.2. Hence, from Theorem 2.1,  $gcat(\mathcal{TC}) = k$ .

# Fixed point theorem for mappings contracting perimeters of triangles and its generalizations

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We establish two generalizations of the fixed point theorem for mappings contracting perimeters of triangles. In the first case we consider these mappings in semimetric spaces with triangle functions introduced by M. Bessenyei and Z. Páles. Such approach allows us to obtain corollaries for different types of semimetric spaces. In the second case we establish the fixed point theorem in ordinary metric spaces for more general class of mappings than mappings contractive perimeters of triangles.

Let  $X$  be a nonempty set. Recall that a mapping  $d: X \times X \rightarrow \mathbb{R}^+$ ,  $\mathbb{R}^+ = [0, \infty)$  is a *metric* if for all  $x, y, z \in X$  the following axioms hold: (i)  $(d(x, y) = 0) \Leftrightarrow (x = y)$ ; (ii)  $d(x, y) = d(y, x)$ ; (iii)  $d(x, y) \leq d(x, z) + d(z, y)$ . The pair  $(X, d)$  is called a *metric space*. If only axioms (i) and (ii) hold then  $d$  is called a *semimetric*. A pair  $(X, d)$ , where  $d$  is a semimetric on  $X$ , is called a *semimetric space*.

In 2017 M. Bessenyei and Z. Páles [1] introduced a definition of a triangle function  $\Phi: \overline{\mathbb{R}}_+^2 \rightarrow \overline{\mathbb{R}}_+$  for a semimetric  $d$ . We use this definition in a slightly different form restricting the domain and the range of  $\Phi$  by  $\mathbb{R}_+^2$  and  $\mathbb{R}^+$ , respectively.

<sup>2</sup>B. Delaunay, Sur la sphère vide. a la mémoire de georges voronoï, Izvestia Akad. Nauk SSSR, Otdelenie Matematicheskii i Estestvennyka Nauk 7 (1934), 793–800.

<sup>3</sup>J.F. Peters, Proximal Voronoï regions, convex polygons, & Leader uniform topology, Advances in Math.: Sci. J. 4 (2015), no. 1, 1–5.

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