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Algebraic and Geometric Methods of Analysis

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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric and topological methods in natural sciences

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On packing and lattice packing of Minkowski-Chebyshev balls

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The Minkowski hypothesis was formulated in [1] and refined in [2, 3, 4]. Regarding the concepts of the geometry of numbers, see [5].

Let

$$D_p = \{(x, y), p > 1\} \subset \mathbb{R}^2 \quad (1)$$

be the 2-dimension region:

$$|x|^p + |y|^p < 1. \quad (2)$$

Let

$$\Delta(p, \sigma) = (\tau + \sigma)(1 + \tau^p)^{-\frac{1}{p}}(1 + \sigma^p)^{-\frac{1}{p}}, \quad (3)$$

be the function defined in the domain

$$\mathcal{M}: \infty > p > 1, 1 \leq \sigma \leq \sigma_p = (2^p - 1)^{\frac{1}{p}}, \quad (4)$$

of the $\{p, \sigma\}$ plane, where σ is some real parameter; here $\tau = \tau(p, \sigma)$ is the function uniquely determined by the conditions

$$A^p + B^p = 1, 0 \leq \tau \leq \tau_p,$$

where

$$A = A(p, \sigma) = (1 + \tau^p)^{-\frac{1}{p}} - (1 + \sigma^p)^{-\frac{1}{p}}, \quad (5)$$

$$B = B(p, \sigma) = \sigma(1 + \sigma^p)^{-\frac{1}{p}}\tau(1 + \tau^p)^{-\frac{1}{p}}, \quad (6)$$

τ_p is defined by the equation

$$2(1 - \tau_p)^p = 1 + \tau_p^p, 0 \leq \tau_p \leq 1. \quad (7)$$

Proposition 1. *The function $\Delta(p, \sigma)$ in region \mathcal{M} determines the moduli space of admissible lattices of the region D_p each of which contains three pairs of points on the boundary of D_p .*

Proposition 2. *Let $\Delta(D_p)$ be the critical determinant of the region $|x|^p + |y|^p < 1$. Let $\Lambda_p^{(0)}$ and $\Lambda_p^{(1)}$ be two D_p -admissible lattices each of which contains three pairs of points on the boundary of D_p and with the property that $(1, 0) \in \Lambda_p^{(0)}$, $(-2^{-1/p}, 2^{-1/p}) \in \Lambda_p^{(1)}$. Under these conditions the lattices are uniquely defined.*

Let $d(\Lambda_p^{(0)}), d(\Lambda_p^{(1)})$ be determinants of the lattices. Let $\Delta_p^{(1)} = \Delta(p, 1) = 4^{-\frac{1}{p}} \frac{1 + \tau_p}{1 - \tau_p}$, $\Delta_p^{(0)} = \Delta(p, \sigma_p) = \frac{1}{2}\sigma_p$.

Proposition 3. $d(\Lambda_p^{(0)}) = \Delta(p, \sigma_p)$, $d(\Lambda_p^{(1)}) = \Delta(p, 1)$.

Remark 4. For example in the case $p = 2$ the lattice $\Lambda_2^{(0)}$ has the determinant $d(\Lambda_2^{(0)}) = \frac{\sqrt{3}}{2}$ and is defined by generators $a_1 = (1, 0), a_2 = (\frac{1}{2}, \frac{\sqrt{3}}{2})$.

Theorem 5. [6]

$$\Delta(D_p) = \begin{cases} \Delta(p, 1), & 1 < p \leq 2, p \geq p_0, \\ \Delta(p, \sigma_p), & 2 \leq p \leq p_0; \end{cases}$$

here p_0 is a real number that is defined unique by conditions $\Delta(p_0, \sigma_p) = \Delta(p_0, 1)$, $2,57 \leq p_0 \leq 2,58$.

Definition 6. In two-dimensional case we will call geometric figures of the form $|x|^p + |y|^p \leq R$, $R > 0$, with $p \geq p_0$ the two-dimensional Minkowski-Chebyshev balls.

In cases of dimension grater then two, when the constant p_0 is unknown, we will call geometric figures of the form $|x_1|^p + |x_2|^p + |x_3|^p + \dots + |x_n|^p \leq R$, $R > 0$, the n -dimensional Minkowski-Chebyshev balls if p is a sufficiently large.

We investigate packing and lattice packing by equal Minkowski-Chebyshev balls of n -dimensional Euclidean spaces and also of corresponding spheres.

Proposition 7. Let \mathbb{Z}^2 be the integer lattice in \mathbb{R}^2 with a point in the origin. Then the density of packing by two-dimensional open Minkowski-Chebyshev balls over the lattice \mathbb{Z}^2 tends to unity as p tends to infinity

Conjecture 8. Let Λ be the integer ($n > 2$)-dimensional lattice in \mathbb{R}^n with a point in the origin. Then the density of packing by n -dimensional open Minkowski-Chebyshev balls over the lattice Λ tends to unity as p tends to infinity

Problem 9. Is there an analogue of Theorem 5 in the case of geometric bodies of the form

$$|x_1|^p + |x_2|^p + |x_3|^p + \dots + |x_n|^p < 1, n > 2,$$

Problem 10. If there exists an analogue of Theorem 5 in the case of geometric bodies of the form

$$|x_1|^p + |x_2|^p + |x_3|^p + \dots + |x_n|^p < 1, n > 2,$$

what is the value of the constant p_0 .

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