



International  
Scientific Conference

# Algebraic and Geometric Methods of Analysis

26-30 may 2020  
Odesa, Ukraine

## LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences

## ORGANIZERS

- Ministry of Education and Science of Ukraine
- Odessa National Academy of Food Technologies
- Institute of Mathematics of the National Academy of Sciences of Ukraine
- Odessa I. I. Mechnikov National University
- Taras Shevchenko National University of Kyiv
- International Geometry Center
- Kyiv Mathematical Society

## PROGRAM COMMITTEE

<b>Chairman: Prishlyak A.</b> ( <i>Kyiv, Ukraine</i> )	<b>Kiosak V.</b> ( <i>Odesa, Ukraine</i> )	<b>Pokas S.</b> ( <i>Odesa, Ukraine</i> )
<b>Balan V.</b> ( <i>Bucharest, Romania</i> )	<b>Kirillov V.</b> ( <i>Odesa, Ukraine</i> )	<b>Polulyakh E.</b> ( <i>Kyiv, Ukraine</i> )
<b>Banakh T.</b> ( <i>Lviv, Ukraine</i> )	<b>Konovenko N.</b> ( <i>Odesa, Ukraine</i> )	<b>Sabitov I.</b> ( <i>Moscow, Russia</i> )
<b>Bolotov D.</b> ( <i>Kharkiv, Ukraine</i> )	<b>Lyubashenko V.</b> ( <i>Kyiv, Ukraine</i> )	<b>Savchenko A.</b> ( <i>Kherson, Ukraine</i> )
<b>Borysenko O.</b> ( <i>Kharkiv, Ukraine</i> )	<b>Maksymenko S.</b> ( <i>Kyiv, Ukraine</i> )	<b>Sergeeva A.</b> ( <i>Odesa, Ukraine</i> )
<b>Cherevko Ye.</b> ( <i>Odesa, Ukraine</i> )	<b>Matsumoto K.</b> ( <i>Yamagata, Japan</i> )	<b>Shelekhov A.</b> ( <i>Tver, Russia</i> )
<b>Fedchenko Yu.</b> ( <i>Odesa, Ukraine</i> )	<b>Mormul P.</b> ( <i>Warsaw, Poland</i> )	<b>Volkov V.</b> ( <i>Odesa, Ukraine</i> )
<b>Karlova O.</b> ( <i>Chernivtsi, Ukraine</i> )	<b>Mykhailyuik V.</b> ( <i>Chernivtsi, Ukraine</i> )	<b>Zarichnyi M.</b> ( <i>Lviv, Ukraine</i> )
	<b>Plachta L.</b> ( <i>Krakov, Poland</i> )	

## ADMINISTRATIVE COMMITTEE

- Egorov B., chairman, rector of the ONAFT;
- Povarova N., deputy chairman, Pro-rector for scientific work of the ONAFT;
- Mardar M., Pro-rector for scientific-pedagogical work and international communications of the ONAFT;
- Fedosov S., Director of the International Cooperation Center of the ONAFT;
- Kotlik S., Director of the P.M. Platonov Educational-scientific institute of computer systems and technologies "Industry 4.0";
- Svytyy I., Dean of the Faculty of Computer Systems and Automation.

## ORGANIZING COMMITTEE

Kirillov V.  
Konovenko N.  
Fedchenko Yu.

Maksymenko S.  
Cherevko Ye.

Osadchuk E.  
Prus A.

ІНТЕРНАЦІОНАЛЬНИЙ ЦЕНТР СПІВРОБІТНИЦТВА

## Dynamics and exact solutions of linear PDEs

**Alexei G. Kushner**

(Lomonosov Moscow State University, GSP-2, Leninskie Gory, Moscow, Russia)

*E-mail:* kushner@physics.msu.ru

**Elena N. Kushner**

(Moscow State Technical University of Civil Aviation, 20 Kronshtadtsky blvd, Moscow, Russia)

*E-mail:* ekushner@ro.ru

**Ruslan I. Matviichuk**

(Lomonosov Moscow State University, GSP-2, Leninskie Gory, Moscow, Russia)

*E-mail:* mathvich@gmail.com

The report presents a new method for constructing exact solutions of the classical linear equations of mathematical physics of parabolic, hyperbolic, elliptic and variable types. The method is a generalization of the theory of finite-dimensional dynamics proposed for evolutionary differential equations [2, 5]. The theory of finite-dimensional dynamics is a natural development of the theory of dynamical systems. Dynamics make it possible to find families that depends on a finite number of parameters among all solutions of PDEs (see [3, 3]).

Consider the following class of second order linear partial differential equations

$$u_{tt} + 2b(x)u_{tx} + c(x)u_{xx} + h(x)u_t + g(x)u_x + f(x) = 0, \quad (1)$$

where  $b, c, h, g, f$  are functions of the class  $C^\infty$ . Such equations are equivalent to the following evolutionary systems

$$\begin{cases} u_t = v, \\ v_t = -2b(x)v_x - c(x)u_{xx} - h(x)v - g(x)u_x - f(y). \end{cases} \quad (2)$$

We call the system of ordinary differential equations of order  $k + 1$

$$\begin{cases} y^{(k+1)} = Y(x, y, z, y', z' \dots, y^{(k)}, z^{(k)}), \\ z^{(k+1)} = Z(x, y, z, y', z' \dots, y^{(k)}, z^{(k)}) \end{cases} \quad (3)$$

a *dynamics* of equation (1) if the vector function

$$(\varphi, \psi) := (z_0, -2b(x)z_1 - c(x)y_2 - h(x)z_0 - g(x)y_1 - f(x))$$

is a generating function of infinitesimal characteristic symmetries of this system [2]. Here  $x, y_0, z_0, y_1, z_1, y_2, z_2$  are canonical coordinates on the space of 2-jets  $J^2(\mathbb{R}^1, \mathbb{R}^2)$ .

**Theorem 1.** *The vector field on  $J^k(\mathbb{R}^1, \mathbb{R}^2)$*

$$S = \varphi \frac{\partial}{\partial y_0} + \psi \frac{\partial}{\partial z_0} + \mathcal{D}(\varphi) \frac{\partial}{\partial y_1} + \mathcal{D}(\psi) \frac{\partial}{\partial z_1} + \dots + \mathcal{D}^k(\varphi) \frac{\partial}{\partial y_k} + \mathcal{D}^k(\psi) \frac{\partial}{\partial z_k} \quad (4)$$

is an infinitesimal characteristic symmetry of system (3) if the following conditions hold:

$$\begin{cases} \mathcal{D}^{k+1}(\varphi) - S(Y) = 0, \\ \mathcal{D}^{k+1}(\psi) - S(Z) = 0. \end{cases} \quad (5)$$

Here

$$\mathcal{D} = \frac{\partial}{\partial x} + y_1 \frac{\partial}{\partial y_0} + z_1 \frac{\partial}{\partial z_0} + \dots + y_k \frac{\partial}{\partial y_{k-1}} + z_k \frac{\partial}{\partial z_{k-1}} + Y \frac{\partial}{\partial y_k} + Z \frac{\partial}{\partial z_k}.$$

Let  $\Gamma^k \subset J^2(\mathbb{R}^1, \mathbb{R}^2)$  be a  $k$ -graph of some solution of system (3) and let  $\Phi_t$  be the shift along the vector field  $S$ . Then the surface  $\Phi_t(\Gamma^k)$  is a  $k$ -graph of a solution of system (2).

**Example 2.** Consider the telegraph equation

$$u_{tt} - u_{xx} = au + bu_t + c, \quad (6)$$

where  $a, b, c$  are constants. This equation admits two types of dynamics:

$$\begin{cases} y_2 = \frac{y_1}{x + \alpha}, \\ z_2 = \frac{z_1}{x + \alpha} \end{cases} \quad (7)$$

and

$$\begin{cases} y_2 = \frac{2b\alpha - (x + \beta)\alpha^2}{4b^2 + 16a - \alpha^2(x + \beta)^2} \times y_1 - \frac{4\alpha}{4b^2 + 16a - \alpha^2(x + \beta)^2} \times z_1, \\ z_2 = -\frac{4a\alpha}{4b^2 + 16a - \alpha^2(x + \beta)^2} \times y_1 - \frac{2b\alpha + \alpha^2(x + \beta)}{4b^2 + 16a - \alpha^2(x + \beta)^2} \times z_1. \end{cases} \quad (8)$$

Here  $\alpha, \beta$  are arbitrary constants. The general solution of equation (7) is

$$\begin{cases} y(x) = C_3 + C_4(x + \alpha)^2, \\ z(x) = C_1 + C_2(x + \alpha)^2, \end{cases} \quad (9)$$

and the general solution of equation (8) is

$$\begin{cases} y(x) = \frac{1}{2}C_2x^2 + C_3x + C_4, \\ z(x) = \frac{1}{8\alpha} (x(C_2\beta - C_3)(2\beta + x)\alpha^2 + (8C_1 + 2bx^2C_2 + 4bC_3x)\alpha - 32 \left(a + \frac{b^2}{4}\right) C_2x). \end{cases} \quad (10)$$

Here  $C_1, \dots, C_4$  are arbitrary constants. Applying the shift transformations  $\Phi_t$  to the obtained general solutions, we obtain particular solutions of equation (6). For example, the function

$$\begin{aligned} u(t, x) = & -1 + \frac{1}{10} \left( \frac{5}{2}x^2 + 5 + (10x + 1 - t)\sqrt{5} \right) e^{-\frac{1}{2}(t\sqrt{5}-1)} + \\ & + \frac{1}{10} \left( \frac{5}{2}x^2 + 5 + (-10x - 1 + t)\sqrt{5} \right) e^{\frac{1}{2}(t\sqrt{5}-1)} \end{aligned} \quad (11)$$

is a solution of equation (6). It corresponds to solution (10) with  $a = b = c = 1$ ,  $\alpha = 1$ ,  $\beta = 0$  and  $C_1 = 0, C_2 = 1, C_3 = 0, C_4 = 0, C_5 = 0$ .

This work is partially supported by Russian Foundation for Basic Research, project 18-29-10013 (A. Kushner).

#### REFERENCES

- [1] Kruglikov B. S., Lychagina O. V. Finite dimensional dynamics for Kolmogorov – Petrovsky – Piskunov equation. *Lobachevskii Journal of Mathematics*, 19: 13–28, 2005.
- [2] Kushner A. G., Matviichuk R.I. Exact solutions of the Burgers – Huxley equation via dynamics. *Journal of Geometry and Physics* 151, 2020.
- [3] Kushner A. G., Matviichuk R.I. Finite Dimensional Dynamics of Evolutionary Equations with Maple. *Differential Geometry, Differential Equations, and Mathematical Physics* Springer (in press).
- [4] Kushner A. G., Lychagin V. V., Rubtsov V. N. Contact geometry and nonlinear differential equations. Cambridge: Cambridge University Press, xxii+496 pp., 2007.
- [5] Lychagin V. V., Lychagina O. V. Finite Dimensional Dynamics for Evolutionary Equations, *Nonlinear Dyn.*, 48: 29–48, 2007.

## ЗМІСТ

<b>G. M. Abdishukurova, A. Ya. Narmanov</b> <i>On the geometry of submersions</i>	<b>3</b>
<b>B. N. Apanasov</b> <i>Hyperbolic 4-cobordisms, Teichmuller spaces and quasiregular mappings in space</i>	<b>5</b>
<b>Aymaz I., Kansu M.</b> <i>Representation of gravi-electromagnetism using matrix algebra</i>	<b>7</b>
<b>V. Bilet, O. Dovgoshey</b> <i>Uniqueness of pretangent spaces at infinity</i>	<b>9</b>
<b>Bolotov D.</b> <i>Foliations of 3-manifolds with small module of mean curvature</i>	<b>10</b>
<b>Bolsinov A. V.</b> <i>On integrability of geodesic flows on 3-dimensional manifolds</i>	<b>11</b>
<b>E. Bonacci</b> <i>Algebraic and geometric questions about the EM helix</i>	<b>12</b>
<b>Borisenko A. A., Sukhorebska D. D.</b> <i>Geodesics on regular tetrahedra in spherical space</i>	<b>13</b>
<b>F. Bulnes</b> <i>Motivic hypercohomology solutions in field theory II</i>	<b>14</b>
<b>I. Denega</b> <i>Estimate of maximum of the products of inner radii of mutually non-overlapping domains</i>	<b>16</b>
<b>A. Dudko, V. Pivovarchik</b> <i>Inverse problem for tree of Stieltjes strings</i>	<b>18</b>
<b>N. Glazunov</b> <i>Formal groups and algebraic cobordism</i>	<b>20</b>
<b>O. Gok</b> <i>A note on tensor product of Archimedean vector lattices</i>	<b>22</b>
<b>E. Gül.</b> <i>Trace Regularization Problem On a Banach Space</i>	<b>24</b>
<b>O. Ye. Hentosh</b> <i>Centrally extended generalization of the superconformal loop Lie algebra and integrable heavenly type systems on supermanifolds</i>	<b>26</b>
<b>B. Hladysh, A. Prishlyak</b> <i>Structure of functions on an oriented 2-manifold with the boundary</i>	<b>28</b>
<b>D. A. Juraev</b> <i>The Cauchy problem for matrix factorizations of the Helmholtz equation in a multidimensional bounded domain</i>	<b>30</b>
<b>A. Kachurovskii</b> <i>Fejer Sums and the von Neumann Ergodic Theorem</i>	<b>31</b>
<b>B. N. Khabibullin, R. R. Muryasov</b> <i>Mixed volumes/areas and distribution of zeros of holomorphic functions</i>	<b>33</b>
<b>B. Klishchuk, R. Salimov</b> <i>On the behavior at infinity of one class of homeomorphisms</i>	<b>35</b>
<b>A. Kravchenko, S. Maksymenko</b> <i>Automorphisms of cellular divisions of 2-sphere induced by functions with isolated critical points</i>	<b>37</b>
<b>A. Kushner, E. Kushner, R. Matviichuk</b> <i>Dynamics and exact solutions of linear PDEs</i>	<b>39</b>