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of analysis»

Book of abstracts



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LIST OF TOPICS

- Algebraic methods in geometry
- Differential geometry in the large
- Geometry and topology of differentiable manifolds
- General and algebraic topology
- Dynamical systems and their applications
- Geometric problems in mathematical analysis
- Geometric and topological methods in natural sciences
- History and methodology of teaching in mathematics

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НТБ ОНАФТ

Artin-Schreier coverings, Galois representations and density Sato-Tate distribution functions

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Let k be a field of characteristic $p > 0$ and let X be a nonsingular projective variety defined over k . In most cases $k = \mathbb{F}_q, q = p^s$ for some positive integer s .

An Artin-Schreier covering of X is a finite morphism $\pi : Y \rightarrow X$ from a normal variety Y onto X such that the field extension $k(Y)/k(X)$ is an Artin-Schreier extension. This extension is defined by the Artin-Schreier equation $y^p - y = f$ and $f \in k(X)$.

Example 1. The equation in affine form $y^p - y = cx + \frac{d}{x}$, $c, d \in \mathbb{F}_p^*$ defines an Artin-Schreier covering of the projective line.

Example 2. The equation in affine form $y^p - y = x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$, $a_i \in \mathbb{F}_p$ defines an Artin-Schreier curve and the respective covering of the projective line.

Proposition 3. See [1, 5]. Let X be reduced absolutely irreducible nonsingular projective algebraic curve of genus g defined over a finite field \mathbb{F}_q . Let $\overline{\mathbb{F}}_q$ be an algebraic closure of \mathbb{F}_q . We may regard X as an algebraic curve over $\overline{\mathbb{F}}_q$. In this case one can attach to X the Jacobean $J(X)$ of X . Formal completion of the variety defines the commutative formal group F of dimension g .

Proposition 4. Under the conditions of the Proposition 3 there is an algebraic construction of the continuation of the Artin-Schreier covering $\pi : Y \rightarrow X$ to the map $\pi' : J(Y) \rightarrow J(X)$.

Let l be a prime such that $l \neq p$. Let $T_l(A)$ be the Tate module of the abelian variety A . In the case of the Jacobean $J(X)$ the Tate module is a free module of the rank $2g$ over l -adic numbers.

Example 5. For any two abelian varieties A_1 and A_2 the canonical homomorphism

$$h_l : \text{Hom}(A_1, A_2) \rightarrow \text{Hom}_{\mathbb{Z}_l}(T_l(A_1), T_l(A_2))$$

is an l -adic representation.

As the formal completion of an abelian variety defines a commutative formal group law (formal group) [2] F , present here some our results about homomorphisms of the groups over local and finite fields.

Let \mathcal{O} be the complete discrete valuation ring of characteristic 0 with maximal ideal $\mathcal{M} = \pi\mathcal{O}$ so the residue field $k = \mathcal{O}/\mathcal{M}$ has characteristic $p > 0$. Let dimensions of formal groups F and G over \mathcal{O} equal respectively n and m . Let $M_{nm}(\mathcal{O})$ be the ring of $n \times m$ matrices over \mathcal{O} .

Proposition 6. Lubin homomorphism $c : \text{Hom}_{\mathcal{O}}(F, G) \rightarrow M_{nm}(\mathcal{O})$ is the injection.

Let $\text{Iso}_R(F, G)$ be the set of isogenies from F to G over a commutative ring R .

Lemma 7. If heights of group laws F and G are different then the set $\text{Iso}_k(F, G)$ is empty.

Let F^* be the reduction of F by $\text{mod } \mathcal{M}$.

Proposition 8. The mapping $*$: $\text{Iso}_{\mathcal{O}}(F, G) \rightarrow \text{Iso}_k(F^*, G^*)$ is the injection if $[p]^*$ is the isogeny.

Definition 9. Let

$$T_p(c, d) = \sum_{x=1}^{p-1} e^{2\pi i \left(\frac{cx + \frac{d}{x}}{p} \right)},$$

$$1 \leq c, d \leq p-1; \quad x, c, d \in \mathbb{F}_p^*$$

be a Kloosterman sum.

Proposition 10. *The Kloosterman sum $T_p(c, d)$ is defined by the Artin-Schreier covering*

$$y^p - y = cx + \frac{d}{x}, \quad c, d \in \mathbb{F}_p^*.$$

By A. Weil estimate $T_p(c, d) = 2\sqrt{p} \cos \theta_p(c, d)$.

There are possible two distributions of angles $\theta_p(c, d)$ on semiinterval $[0, \pi)$:

- a) p is fixed and c and d varies over \mathbb{F}_p^* ; what is the distribution of angles $\theta_p(c, d)$ as $p \rightarrow \infty$;
- b) c and d are fixed and p varies over all primes not dividing c and d .

For the case a) N. Katz [3] and A. Adolphson [4] for few others sums proved that θ are distributed on $[0, \pi)$ with the Sato-Tate density $\frac{2}{\pi} \sin^2 t$.

Conjecture 11. *In the case b) when p varies over all primes then angles $\theta_p(1, 1)$ are distributed on the interval $[0, \pi)$ with the Sato-Tate density $\frac{2}{\pi} \sin^2 t$.*

REREFENCES

- [1] David Mumford. *Abelian Varieties*. *Tata Institute of Fundamental Research Publications*, 2012, vol. 13, 263 p.
- [2] Michiel Hazewinkel. *Formal Groups and Applications*, Providence, Rhode Island, Academic Press, 2012.
- [3] Nicholas Katz. *Gauss sums, Kloosterman sums, and monodromy groups*. Princeton university press, 1988, 246 p.
- [4] Alan Adolphson. *On the distribution of angles of Kloosterman sums*. *Journ. rein. angew. Math.*, 1989, vol. 395, P. 214–220.
- [5] Nikolaj Glazunov. *Extremal forms and rigidity in arithmetic geometry and in dynamics*, *Chebyshevskii Sbornik*, 2015, vol.16, no. 3, P. 124–146.

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